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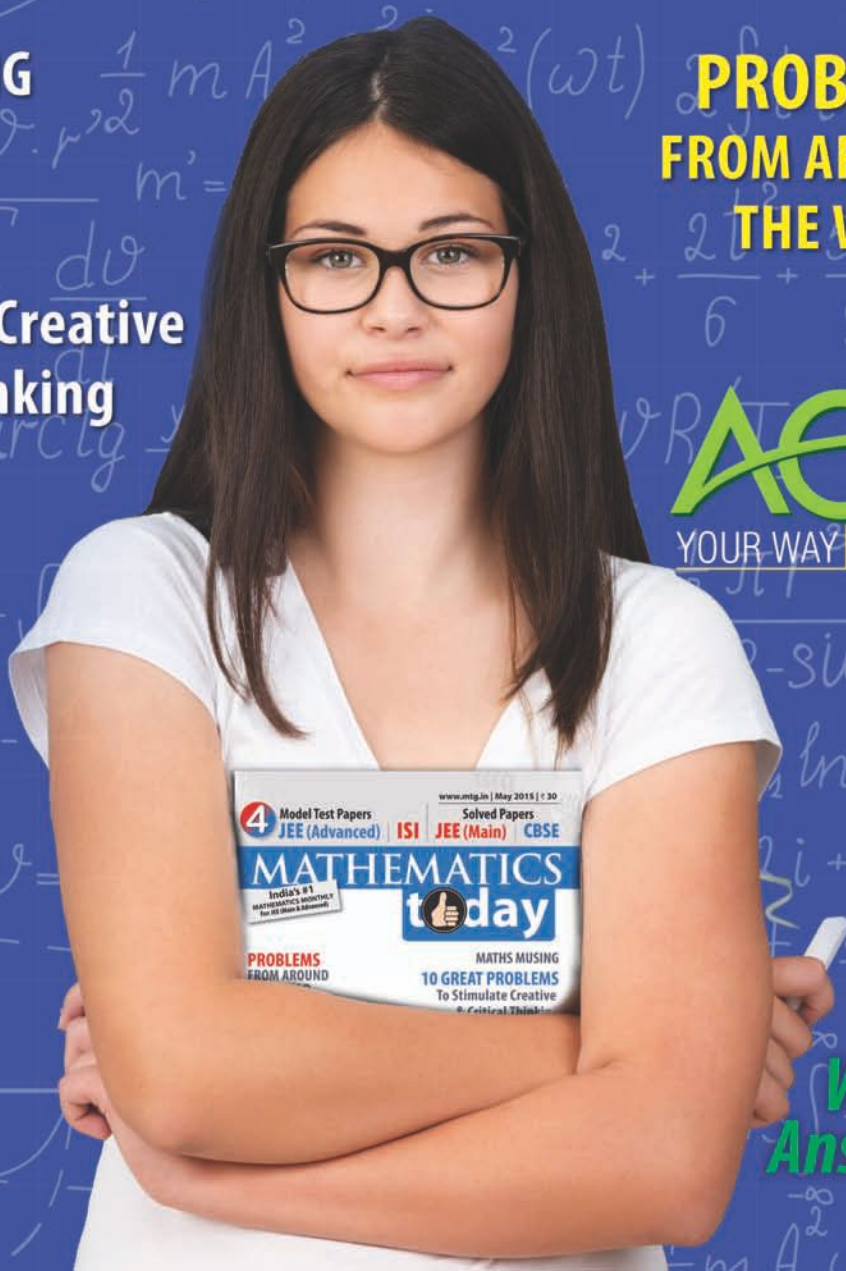

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editorial

Hints to Sugar-coat Maths

Persons can be classified as either those who love mathematics or those who are afraid of Mathematics. The latter category also detest mathematics because of their inherent fear. Mathematics and grammar have parallel methods of learning. Grammar is defined as the art of the right use of language by grammatical rules. One learns the abstract rules first, learns by note the various tenses and the application part comes later.

The problem of application of grammar is avoided by successful authors of books on languages by first using the language in human situations with small doses of grammar at the end of the lesson. Should we not first start by applications and give the rules slowly pushing them so that the student learns the rules from application. Every branch of mathematics should be treated with applications first with the theory hidden like nuts in sweets. After the students have taken a taste for calculations, then the rules should be given as a secret.

The examples should be from as many branches of knowledge as possible so that students are kept on their toes. Maths should not be given to the students as the old ayurvedic corrections but sweet end with honey or like the sugar-coated pills of homeopathy. We can assure the students that the study of maths, physics or biology is more interesting than reading the detective novels and you can have more fun.

With best wishes and good luck,

Anil Ahlawat
Editor

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today with the suggestion of Shri Mahabir Singh. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.
During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.
Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

Prof. Dr. Ramanaiah Gundala, Former Dean of Science and Humanities, Anna University, Chennai

PROBLEM Set 150

JEE MAIN

1. Let S be the sum of the first 2015 terms of the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ... where n occurs as n terms. The sum of the digits of S is

(a) 21 (b) 22 (c) 23 (d) 24

2. Two circles S_1 and S_2 pass through the points $(0, a)$ and $(0, -a)$. The line $y = mx+c$ is a tangent to the two circles. If S_1 and S_2 are orthogonal, then

(a) $a^2(m^2 + 1) = c^2$ (b) $a^2(m^2 + 2) = c^2$
(c) $c^2(m^2 + 1) = a^2$ (d) $c^2(m^2 + 2) = a^2$

3. $OABC$ is a tetrahedron with all the edges are of unit length. The shortest distance between the edges OA and BC is

(a) $\sqrt{\frac{2}{3}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

4. If $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, then $A^{-1} + A^2 =$

(a) $I + 3A$ (b) $I - 3A$
(c) $3A - I$ (d) $2I$

5. An ellipse has its axes along the coordinate axes and passes through the points (1, 1) and $\left(\frac{1}{\sqrt{2}}, \frac{\sqrt{5}}{2}\right)$. Its eccentricity is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{2}{3}$ (d) $\sqrt{\frac{2}{3}}$

JEE ADVANCED

6. $i^i =$

(a) $e^{\pi/2}$ (b) $e^{3\pi/2}$ (c) $e^{-\pi/2}$ (d) $e^{-3\pi/2}$

COMPREHENSION

If $I = \int_0^{\pi/2} \sin^{-1}(c \cos x)\, dx$, $c < 1 = \sum_{n=1}^{\infty} \frac{c^{2n-1}}{(2n-1)^2}$, then for $c = 1$

7. $\sum_{n=1}^{\infty} \frac{1}{n^2} =$

(a) $\frac{\pi^2}{12}$ (b) $\frac{\pi^2}{8}$ (c) $\frac{\pi^2}{6}$ (d) $\frac{\pi^2}{16}$

8. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

(a) $\frac{\pi^2}{12}$ (b) $\frac{\pi^2}{8}$ (c) $\frac{\pi^2}{6}$ (d) $\frac{\pi^2}{16}$

INTEGER MATCH

9. A fair coin is tossed 10 times. Let $\frac{m}{n}$, in the lowest terms, be the probability that heads never occur on consecutive tosses. Then m is

MATCHING LIST

10. Let C_n be the coefficient of x^n in the expansion of $(1+x+x^2)^n$.

Column-I		Column-II	
P.	If $n = 3$, the last digit of C_n is	1.	1
Q.	If $n = 4$, the last digit of C_n is	2.	3
R.	If $n = 5$, the last digit of C_n is	3.	7
S.	If $n = 7$, the last digit of C_n is	4.	9

	P	Q	R	S
(a)	1	2	3	4
(b)	2	3	4	1
(c)	3	1	2	4
(d)	3	4	1	2

See Solution set of Maths Musing 149 on page no. 81

Prof. Ramanaiah is the author of MTG JEE(Main & Advanced) Mathematics series

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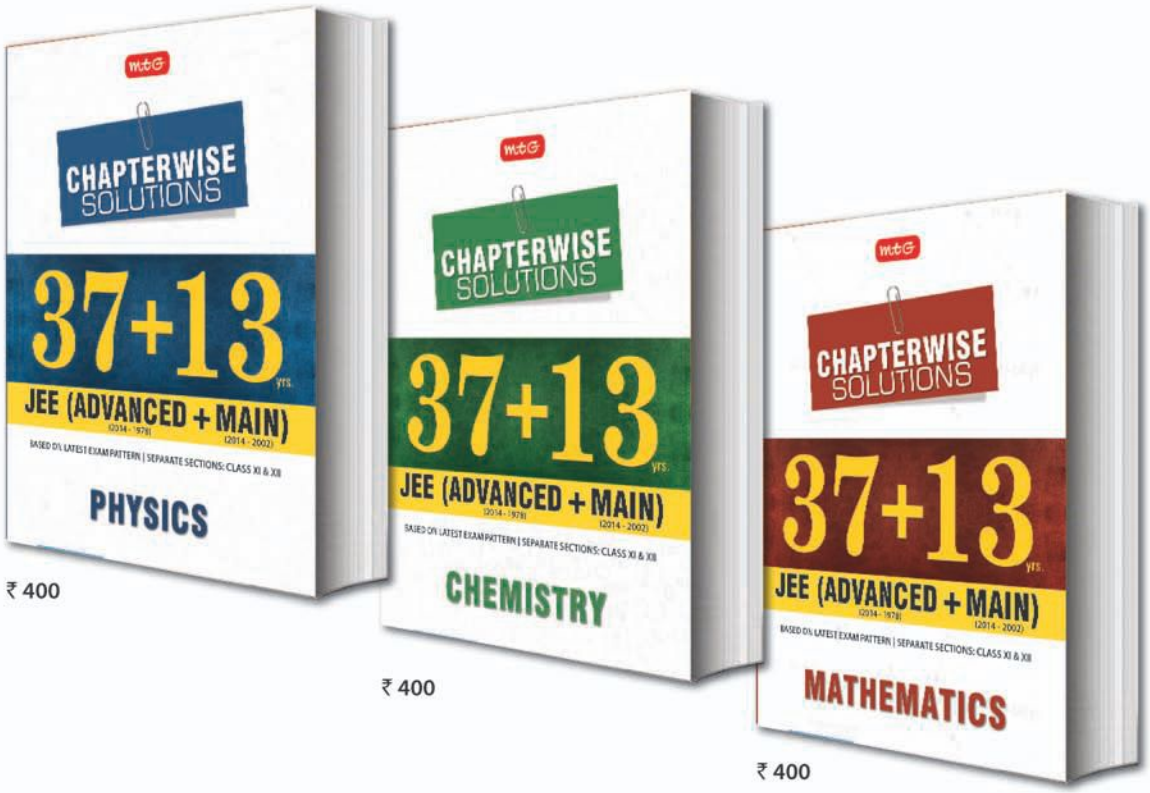

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JEE WORK OUTS

SECTION-I

SINGLE CORRECT OPTION

This section contains 8 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONLY ONE is correct. [3 marks for correct answer and –1 for wrong answer]

1. On a railway there are 10 stations. The number of types of tickets required in order that it may be possible to book a passenger from every station to every other is

(a) $\frac{10!}{8!}$ (b) $10!\,2!$ (c) $\frac{10!}{2!}$ (d) $\frac{10!}{8!\,2!}$

2. If the roots of $ax^2 + bx + c = 0$ are of the form $\frac{m}{m-1}$ and $\frac{m+1}{m}$ then the value of $(a + b + c)^2$ is

(a) $b^2 - 2ac$ (b) $2b^2 - ac$
(c) $b^2 - 4ac$ (d) $2(b^2 - 2ac)$

3. If A and B are two square matrices of order 3×3 which satisfy $AB = A$ and $BA = B$, then $(A + B)^7$ is

(a) $7(A + B)$ (b) $7 \cdot I_{3\times 3}$
(c) $64(A + B)$ (d) $128\, I_{3\times 3}$

4. If a_1, a_2, a_3, a_4, a_5 are in H.P., then $a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5$ is equal to

(a) $2a_1a_5$ (b) $3a_1a_5$ (c) $8a_1a_5$ (d) $4a_1a_5$

5. The maximum value of $(\sin\alpha_1)(\sin\alpha_2) \ldots (\sin\alpha_n)$ under the restrictions $0 \le \alpha_1, \alpha_2, \ldots \alpha_n < \pi/2$ and $(\tan\alpha_1)(\tan\alpha_2) \ldots (\tan\alpha_n) = 1$ is

(a) $\frac{1}{2^n}$ (b) $\frac{1}{2n}$ (c) $\frac{1}{2^{n/2}}$ (d) 1

6. In a conference 10 speakers are to give their speaches one after another. Find the probability of the event if S_1 speaks before S_2 and S_2 speaks before S_3 and the remaining 7 speakers have no objection to speak at any number?

(a) 1/6 (b) 1/3 (c) 1/45 (d) 3/10

7. i^i is a

(a) complex number
(b) purely imaginary number
(c) real number (d) none of these

8. If k and K are minimum and maximum values of $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ respectively, then

(a) $k = \frac{\pi}{4}, K = \frac{3\pi}{4}$ (b) $k = 0, K = \pi$

(c) $k = \frac{\pi}{2}, K = \pi$ (d) not defined

SECTION-II

MULTIPLE CORRECT OPTION

This section contains 4 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct. [4 marks for correct answer and –1 for wrong answer]

9. If $f(x)$ and $g(x)$ are functions such that $f(x + y) = f(x) \cdot g(y) + g(x) \cdot f(y)$ then

$$\begin{vmatrix} f(\alpha) & g(\alpha) & f(\alpha+\theta) \\ f(\beta) & g(\beta) & f(\beta+\theta) \\ f(\gamma) & g(\gamma) & f(\gamma+\theta) \end{vmatrix}$$ is independent of

(a) α (b) β (c) γ (d) θ

10. Sum of the roots of the equation $(x + 1) = 2\log_2(2^x + 3) - 2\log_4(1980 - 2^{-x})$ is greater than

(a) 2 (b) 3 (c) 4 (d) 5

11. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$, then

(a) $P(B/A) = \frac{P(B)}{P(A)}$

(b) $P(A^C \cup B^C) = P(A^C) + P(B^C)$

(c) $P((A \cup B)^C) = P(A^C)\, P(B^C)$

(d) $P(A/B) = P(A)$

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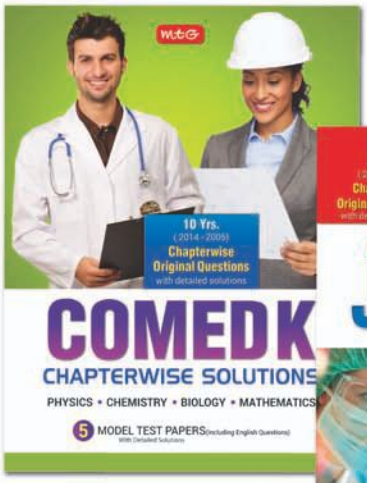

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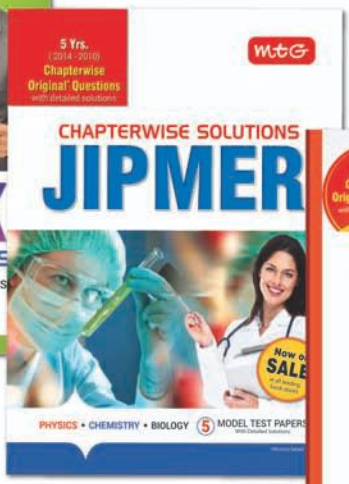

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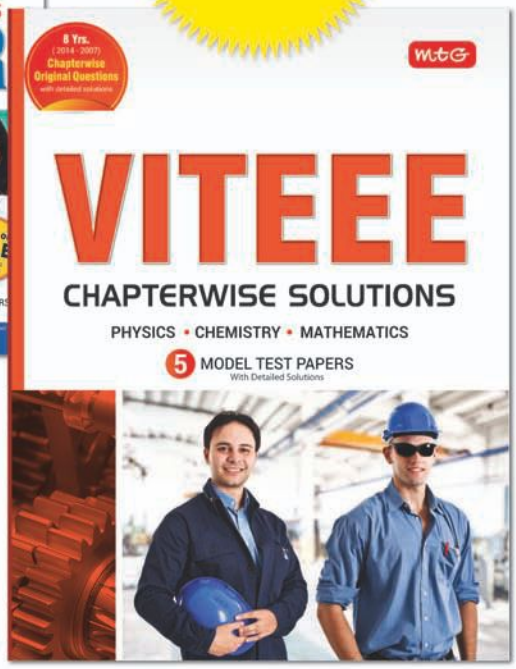

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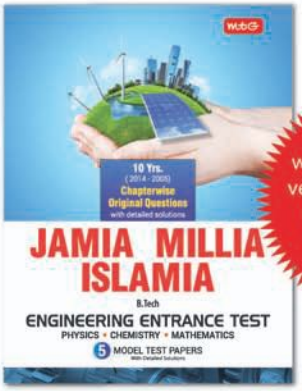

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12. The origin and roots of the equation $z^2 + pz + q = 0$ form an equilateral triangle if

(a) $p^2 = q$ (b) $p^2 = 3q$

(c) $q^2 = 3p$ (d) $q^2 = p$

SECTION-III

LINKED COMPREHENSION TYPE

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which one or more than one can be correct. [4 marks for correct answer and –1 for wrong answer]

Paragraph for Question Nos. 13 to 15

Let p be a prime number and n be a positive integer, then exponent of p is $n!$ is denoted by $E_p(n!)$ and is given by

$$E_P(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots + \left[\frac{n}{p^k}\right]$$

where $p^k < n < p^{k+1}$ and $[x]$ denotes the integral part of x.

If we isolate the power of each prime contained in any number N, then N can be written as

$N = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3}$ .... where p_1, p_2, p_3 ... prime numbers and $\alpha_1, \alpha_2, \alpha_3$, ... are natural numbers.

13. The numbers of zeroes at the end of 108! is

(a) 24 (b) 25 (c) 26 (d) 21

14. The number of prime numbers among the numbers

$101! + 2$, $101! + 3$, $101! + 4$, ..., $101! + 101$ is

(a) 31 (b) 29

(c) 53 (d) none of these

15. The last non-zero digit in 20! must be equal to

(a) 2 (b) 3 (c) 4 (d) 9

Paragraph for Question Nos. 16 to 18

The numbers 1, 3, 6, 10, 15, 21, 28, ... are called triangular numbers. Let t_n denotes the n^{th} triangular number then it can be observed that $t_1 = 1$, $t_2 = 3$, $t_n = t_{n-1} + n$.

16. t_{100} must be equal to

(a) 5050 (b) 5151

(c) 5252 (d) none of these

17. If m is n^{th} triangular number then

(a) $n = \dfrac{\sqrt{1+8m}+1}{2}$ (b) $n = \dfrac{\sqrt{1+8m}-1}{2}$

(c) $n = \dfrac{\sqrt{1+4m}-1}{2}$ (d) none of these

18. The number of positive integers lying between t_{50} and t_{51} must be

(a) 50 (b) 51

(c) 52 (d) none of these

SECTION-IV

MATRIX-MATCH TYPE

This section contains 2 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled. [8 marks for correct answer and no negative marking for wrong answer and each row 2 marks]

19. Match the following:

	Column I		Column II
(A)	If n be the number of ways in which 12 different books can be distributed equally among 3 persons, then $\dfrac{(4!)^4 n}{12!}$ is divisible by	p.	4
(B)	If λ be the number of integral solutions of $x + y + z = 15$ such that $x \geq 1$, $y \geq 2$ and $z \geq 3$ then λ is divisible by	q.	6
(C)	If λ be maximum number of points of intersection of 8 straight lines then λ is divisible by	r.	12
(D)	A car will hold 2 persons in the front seat and 1 in the near seat. If among 6 persons only 2 can drive, the number of ways in which car can be filled is λ then λ is divisible by	s.	5

20. If in a triangle $2a^2 + 4b^2 + c^2 = 4ab + 2ac$, then match the following:

	Column I		Column II
(A)	cos$A$	p.	1/4
(B)	cos$B$	q.	7/8
(C)	cos$C$	r.	0
(D)	sin$(A - C)$	s.	1

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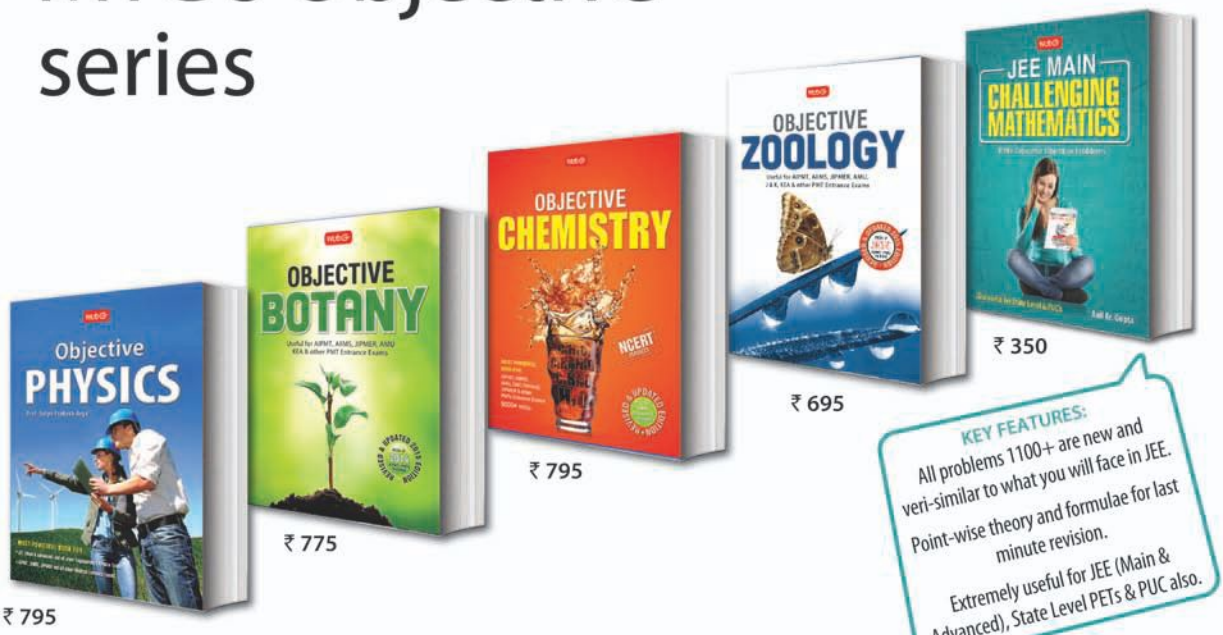

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SECTION-I

SINGLE CORRECT OPTION

This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct. [3 marks for correct answer and –1 for wrong answer].

1. The remainder when 5^{97} is divided by 52 is
(a) 2 (b) 3 (c) 5 (d) 10

2. The number of real solutions of (x, y), where $y = |\sin x|$, $y = \cos^{-1}(\cos x)$, $-2\pi \le x \le 2\pi$, is
(a) 2 (b) 1 (c) 3 (d) 4

3. In ΔABC, $\angle A = \angle C = \pi/6$ and radius of incircle is r, if radius of circumscribing circle $R = 4$ then r is equal to
(a) $4\sqrt{3}-6$ (b) $4\sqrt{3}+6$
(c) $2\sqrt{3}-2$ (d) $2\sqrt{3}+2$

4. If $2x = -1+\sqrt{3}i$, then the value of $(1 - x^2 + x)^6 - (1 - x + x^2)^6 =$
(a) 32 (b) –64 (c) 64 (d) 0

SECTION-II

MULTIPLE CORRECT OPTION

This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct. [4 marks for correct answer and –1 for wrong answer]

5. Let $\{\Delta_1, \Delta_2, \Delta_3, ..., \Delta_k\}$ be the set of third-order determinants that can be made with the distinct non-zero real numbers $a_1, a_2, a_3, ..., a_9$. Then
(a) $k = 9!$ (b) $\sum_{i=1}^{k} \Delta_i = 0$
(c) at least one $\Delta_i = 0$ (d) none of these

6. The quadratic equation $x^2 - 2x - \lambda = 0$, $\lambda \neq 0$,
(a) cannot have a real roots if $\lambda < -1$
(b) can have a rational root if λ is a perfect square of an integer
(c) cannot have an integral root if $n^2 - 1 < \lambda < n^2 + 2n$ where $n = 0, 1, 2, 3, ...$
(d) none of these

7. Given the set of four real numbers. First three numbers are in G.P. and last three numbers are in A.P. The common difference of A.P. is 6. First and fourth terms are same. According to the given data,
(a) fourth term = 4 (b) fourth term = 8
(c) sum of 4 nos. = 14 (d) sum of 4 nos. = 16

8. If $x + y + z = 5$ and $xy + yz + zx = 3$; $x, y, z \in R$ then for x,
(a) largest value = $\frac{13}{3}$
(b) largest value = $\frac{10}{3}$
(c) least value = –1 (d) least value = 5

9. In the expansion of $\left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$,
(a) the number of irrational terms = 19
(b) middle term is irrational
(c) the number of irrational terms = 15
(d) 9^{th} term is rational

SECTION-III

MATRIX-MATCH TYPE

This section contains 2 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled. [8 marks for correct answer and no negative marking for wrong answer and each row 2 marks]

10. Match the following:

	Column I		Column II
(A)	Any two small squares on a chess board are chosen at random. Probability that they have a common side is	p.	1/10
(B)	One of ten keys opens the door. If we try the keys one after another, then the probability that the door is opened on the tenth attempt is	q.	0
(C)	The probability of obtaining no head in an infinite sequence of independent tosses of a coin is	r.	$\frac{7}{144}$
(D)	Two squares are chosen at random from the small squares drawn on a chess board. The chance that the two squares chosen have exactly one corner in common is	s.	1/18

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11. Match the following:

Column I		Column II	
(A)	M_r is defined as : $M_r = \begin{bmatrix} r-1 & \frac{1}{r} \\ 1 & \frac{1}{(r-1)^2} \end{bmatrix}$ and $\lvert M_r \rvert$ is the corresponding determinant value of M_r, then $\lim_{n\to\infty}(\lvert M_2 \rvert + \lvert M_3 \rvert + ... + \lvert M_n \rvert) =$	p.	0
(B)	If $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$ then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$	q.	4
(C)	If $A = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$ and a matrix C is defined as $C = (BAB^{-1})(B^{-1}A^TB)$, where $\lvert B \rvert \neq 0$, then $\lvert C \rvert = k^2$, $(k \in N)$ where $k =$	r.	2
(D)	If $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ and $A^4 = -\lambda I$, then λ equals	s.	1

SECTION-IV

INTEGER ANSWER TYPE

This section contains 8 questions. The answer to each of the questions is a single–digit integer, ranging from 0 to 9. [4 marks for correct answer and –1 for wrong answer]

12. Find the number of roots of the equation $z^{10} = 1$ satisfying $\lvert \arg z \rvert < (\pi/2)$.

13. Find the number of quadratic equations which remain unchanged by squaring their roots.

14. $A(0, 0)$, $B(4, 2)$ and $C(6, 0)$ are the vertices of a triangle ABC and BD is its altitude. The line through D parallel to the side AB intersects the side BC at a point E. Find the product of areas of ΔABC and ΔBDE.

15. A coin is tossed twice and the four possible outcomes are assumed to be equally likely. If A is the event, 'both head and tail have appeared', and B be the event, 'at most one tail is observed', then $5[P(B/A)]$ is

16. The sum of the roots of the equation $4\sin^3\left(\frac{\pi}{2}+x\right) - 4\cos^2 x - \cos(\pi + x) - 1 = 0$ in the interval $[0, 4\pi]$ is $p\pi$. Find p.

17. In ΔABC, $\angle A = 30°$ and $\angle C = 105°$. Find k such that $k = \left(\frac{c^2 - b^2}{a^2}\right)^2$.

18. Find the number of integral values of λ if $(\lambda, 2)$ is an interior point of ΔABC formed by $x + y = 4$, $3x - 7y = 8$, $4x - y = 31$.

19. The equation $ax^2 + bx + 6 = 0$ does not have two distinct real roots, then the least value of $3a + b + 3$ is

ANSWERS

PAPER-1

1. (a) **2.** (c) **3.** (c) **4.** (d) **5.** (c)
6. (a) **7.** (c) **8.** (a) **9.** (a, b, c, d)
10. (a, b) **11.** (c, d) **12.** (b) **13.** (b) **14.** (d)
15. (c) **16.** (a) **17.** (b) **18.** (a)
19. (A) → (p, q, r); (B) → (s); (C) → (p); (D) → (p, s)
20. (A) → (p) ; (B) → (q) ; (C) → (p) ; (D) → (r)

PAPER-2

1. (c) **2.** (c) **3.** (a) **4.** (d) **5.** (a, b)
6. (a, c) **7.** (b, c) **8.** (a, c) **9.** (a, b, d)
10. (A) → (s) ; (B) → (p) ; (C) → (q) ; (D) → (r)
11. (A) → (s) ; (B) → (r) ; (C) → (q) ; (D) → (q)
12. (5) **13.** (4) **14.** (8) **15.** (5) **16.** (6)
17. (3) **18.** (1) **19.** (1)

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WBJEE SOLVED PAPER 2015

CATEGORY – I (Q1 to Q60)

Each question has one correct option and carries 1 mark, for wrong answer 1/4 mark will be deducted

1. In a certain town, 60% of the families own a car, 30% own a house and 20% own both a car and a house. If a family is randomnly chosen, what is the probability that this family owns a car or a house but not both?

(a) 0.5 (b) 0.7 (c) 0.1 (d) 0.9

2. The letters of the word COCHIN are permuted and all the permutations are arranged in alphabetical order as in English dictionary. The number of words that appear before the word COCHIN is

(a) 360 (b) 192 (c) 96 (d) 48

3. Let $f : R \to R$ be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$. Then the value of $f(\log_e 5)$ is

(a) 0 (b) 2 (c) 5 (d) 3

4. The value of $\lim_{x\to 2} \int_2^x \frac{3t^2}{x-2} dt$ is

(a) 10 (b) 12 (c) 8 (d) 16

5. If $\cot\frac{2x}{3} + \tan\frac{x}{3} = \text{cosec}\frac{kx}{3}$, then the value of k is

(a) 1 (b) 2 (c) 3 (d) –1

6. If $\theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, then the value of

$\sqrt{4\cos^4\theta + \sin^2 2\theta} + 2\cot\theta\, 2\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$ is

(a) $-2\cot\theta$ (b) $2\cot\theta$
(c) $2\cos\theta$ (d) $2\sin\theta$

7. The number of real solutions of the equation $(\sin x - x)(\cos x - x^2) = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

8. The value of λ, such that the following system of equations has no solution, is

$x - 2y + z = -4$
$2x - y - 2z = 2$
$x + y + \lambda z = 4$

(a) 3 (b) 1 (c) 0 (d) –3

9. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$

Then $f(100)$ is equal to

(a) 0 (b) 1 (c) 100 (d) 10

10. Which of the following is not always true?

(a) $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$ if $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

(b) $|\vec{a}+\lambda\vec{b}| \geq |\vec{a}|$ for all $\lambda \in R$ if $\vec{a}$ and $\vec{b}$ are perpendicular to each other

(c) $|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2\left(|\vec{a}|^2 + |\vec{b}|^2\right)$

(d) $|\vec{a}+\lambda\vec{b}| \geq |\vec{a}|$ for all $\lambda \in R$ if $\vec{a}$ is parallel to $\vec{b}$

11. If the four points with position vectors $-2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\hat{k}$, $\hat{j}-\hat{k}$ and $\lambda\hat{j}+\hat{k}$ are coplanar then λ =

(a) 1 (b) 2 (c) –1 (d) 0

12. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \frac{x^4}{8} + \ldots\right) = \frac{\pi}{6}$ where $|x| < 2$ then the value of x is

(a) 2/3 (b) 3/2 (c) –2/3 (d) –3/2

13. The area of the region bounded by the curve $y = x^3$, its tangent at (1, 1) and x-axis is

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{2}{17}$ (d) $\frac{2}{15}$

14. If $\log_{0.2}(x-1) > \log_{0.04}(x+5)$ then

(a) $-1 < x < 4$ (b) $2 < x < 3$
(c) $1 < x < 4$ (d) $1 < x < 3$

By : **Anil Kr. Gupta (AKG Classes), ASANSOL, Kolkata** (W.B). Mob.: 09832230099

15. The number of real roots of equation $\log_e x + ex = 0$ is

(a) 0 (b) 1 (c) 2 (d) 3

16. For all real values of a_0, a_1, a_2, a_3 satisfying $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} = 0$, the equation $a_0 + a_1x + a_2x^2 + a^3x^3 = 0$ has a real root in the interval

(a) $[0, 1]$ (b) $[-1, 0]$ (c) $[1, 2]$ (d) $[-2, -1]$

17. Let $f: R \to R$ be defined as

$f(x) = \begin{cases} 0, & x \text{ is irrational} \\ \sin|x|, & x \text{ is rational} \end{cases}$. Then which of the following is true?

(a) f is discontinuous for all x

(b) f is continuous for all x

(c) f is discontinuous at $x = k\pi$, where k is an integer

(d) f is continuous at $x = k\pi$, where k is an integer.

18. If the vertex of the conic $y^2 - 4y = 4x - 4a$ always lies between the straight lines $x + y = 3$ and $2x + 2y - 1 = 0$ then

(a) $2 < a < 4$ (b) $-\frac{1}{2} < a < 2$

(c) $0 < a < 2$ (d) $-\frac{1}{2} < a < \frac{3}{2}$

19. Number of intersecting points of the conic $4x^2 + 9y^2 = 1$ and $4x^2 + y^2 = 4$ is

(a) 1 (b) 2 (c) 3 (d) 0

20. The value of λ for which the straight line $\frac{x-\lambda}{3} = \frac{y-1}{2+\lambda} = \frac{z-3}{-1}$ may lie on the plane $x - 2y = 0$ is

(a) 2 (b) 0

(c) $-1/2$ (d) there is no such λ

21. If $f: [0, \pi/2) \to R$ is defined as

$$f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$$

Then the range of f is

(a) $(2, \infty)$ (b) $(-\infty, -2]$

(c) $[2, \infty)$ (d) $(-\infty, 2]$

22. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ equals

(a) $2AB$ (b) $2BA$ (c) $A + B$ (d) AB

23. If ω is an imaginary cube root of unity, then the value of the determinant $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega+\omega^2 & \omega & -\omega^2 \end{vmatrix}$ is

(a) -2ω (b) $-3\omega^2$ (c) -1 (d) 0

24. The value of $2\cot^{-1}\frac{1}{2} - \cot^{-1}\frac{4}{3}$ is

(a) $-\frac{\pi}{8}$ (b) $\frac{3\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

25. If the point $(2\cos\theta, 2\sin\theta)$, for $\theta \in (0, 2\pi)$ lies in the region between the lines $x + y = 2$ and $x - y = 2$ containing the origin, then θ lies in

(a) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$ (b) $[0, \pi]$

(c) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ (d) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

26. Number of points having distance $\sqrt{5}$ from the straight line $x - 2y + 1 = 0$ and a distance $\sqrt{13}$ from the line $2x + 3y - 1 = 0$ is

(a) 1 (b) 2 (c) 4 (d) 5

27. Let a, b, c, d be any four real numbers. Then $a^n + b^n = c^n + d^n$ holds for any natural number n if

(a) $a + b = c + d$ (b) $a - b = c - d$

(c) $a + b = c + d$, $a^2 + b^2 = c^2 + d^2$

(d) $a - b = c - d$, $a^2 - b^2 = c^2 - d^2$

28. If α, β are the roots of $x^2 - px + 1 = 0$ and γ is a root of $x^2 + px + 1 = 0$, then $(\alpha + \gamma)(\beta + \gamma)$ is

(a) 0 (b) 1 (c) -1 (d) p

29. Number of irrational terms in the binomial expansion of $(3^{1/5} + 7^{1/3})^{100}$ is

(a) 90 (b) 88 (c) 93 (d) 95

30. The quadratic expression $(2x + 1)^2 - px + q \neq 0$ for any real x if

(a) $p^2 - 16p - 8q < 0$ (b) $p^2 - 8p + 16q < 0$

(c) $p^2 - 8p - 16q < 0$ (d) $p^2 - 16p + 8q < 0$

31. Let $f: R \to R$ be defined as $f(x) = \frac{x^2 - x + 4}{x^2 + x + 4}$. Then the range of the function $f(x)$ is

(a) $\left[\frac{3}{5}, \frac{5}{3}\right]$ (b) $\left(\frac{3}{5}, \frac{5}{3}\right)$

(c) $\left(-\infty, \frac{3}{5}\right) \cup \left(\frac{5}{3}, \infty\right)$ (d) $\left[-\frac{5}{3}, -\frac{3}{5}\right]$

32. The least value of $2x^2 + y^2 + 2xy + 2x - 3y + 8$ for real numbers x and y is

(a) 2 (b) 8 (c) 3 (d) 1

33. Let $f : [-2, 2] \to R$ be a continuous function such that $f(x)$ assumes only irrational values. If $f(\sqrt{2}) = \sqrt{2}$, then

(a) $f(0) = 0$ (b) $f(\sqrt{2}-1) = \sqrt{2}-1$

(c) $f(\sqrt{2}-1) = \sqrt{2}+1$ (d) $f(\sqrt{2}-1) = \sqrt{2}$

34. The minimum value of $\cos\theta + \sin\theta + \dfrac{2}{\sin 2\theta}$ for $\theta \in (0, \pi/2)$ is

(a) $2+\sqrt{2}$ (b) 2

(c) $1+\sqrt{2}$ (d) $2\sqrt{2}$

35. The value of $\left(\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{64} + \left(\dfrac{1-\sqrt{3}i}{1+\sqrt{3}i}\right)^{64}$ is

(a) 0 (b) –1 (c) 1 (d) i

36. Find the maximum value of $|z|$ when $\left|z - \dfrac{3}{z}\right| = 2$, z being a complex number.

(a) $1+\sqrt{3}$ (b) 3

(c) $1+\sqrt{2}$ (d) 1

37. Given that x is a real number satisfying $\dfrac{5x^2 - 26x + 5}{3x^2 - 10x + 3} < 0$, then

(a) $x < \dfrac{1}{5}$ (b) $\dfrac{1}{5} < x < 3$

(c) $x > 5$ (d) $\dfrac{1}{5} < x < \dfrac{1}{3}$ or $3 < x < 5$

38. Let $x_n = \left(1-\dfrac{1}{3}\right)^2\left(1-\dfrac{1}{6}\right)^2\left(1-\dfrac{1}{10}\right)^2 \ldots \left(1-\dfrac{1}{\frac{n(n+1)}{2}}\right)^2$

$n \geq 2$. Then the value of $\lim\limits_{n\to\infty} x_n$ is

(a) 1/3 (b) 1/9 (c) 1/81 (d) 0

39. The variance of first 20 natural numbers is

(a) 133/4 (b) 279/12 (c) 133/2 (d) 399/4

40. A fair coin is tossed a fixed number of times. If the probability of getting exactly 3 heads equals the probability of getting exactly 5 heads, then the probability of getting exactly one head is

(a) 1/64 (b) 1/32 (c) 1/16 (d) 1/8

41. If the letters of the word PROBABILITY are written down at random in a row, the probability that two B's are together is

(a) 2/11 (b) 10/11

(c) 3/11 (d) 6/11

42. The least positive value of t so that the lines $x = t + \alpha$, $y + 16 = 0$ and $y = \alpha x$ are concurrent is

(a) 2 (b) 4 (c) 16 (d) 8

43. If in a triangle ABC, $a^2\cos^2 A - b^2 - c^2 = 0$, then

(a) $\dfrac{\pi}{4} < A < \dfrac{\pi}{2}$ (b) $\dfrac{\pi}{2} < A < \pi$

(c) $A = \dfrac{\pi}{2}$ (d) $A < \dfrac{\pi}{4}$

44. $\{x \in R : |\cos x| \geq \sin x\} \cap \left[0, \dfrac{3\pi}{2}\right] =$

(a) $\left[0, \dfrac{\pi}{4}\right] \cup \left[\dfrac{3\pi}{4}, \dfrac{3\pi}{2}\right]$

(b) $\left[0, \dfrac{\pi}{4}\right] \cup \left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$

(c) $\left[0, \dfrac{\pi}{4}\right] \cup \left[\dfrac{5\pi}{4}, \dfrac{3\pi}{2}\right]$

(d) $\left[0, \dfrac{3\pi}{2}\right]$

45. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\dfrac{\pi}{4} \leq x \leq \dfrac{\pi}{4}$ is

(a) 0 (b) 2 (c) 1 (d) > 2

46. Let $x_1, x_2, \ldots, x_{15}$ be 15 distinct numbers chosen from 1, 2, 3, ..., 15. Then the value of $(x_1 - 1)(x_2 - 1)(x_3 - 1) \ldots (x_{15} - 1)$ is

(a) always ≤ 0 (b) 0

(c) always even (d) always odd

47. Let $[x]$ denote the greatest integer less than or equal to x. Then the value of α for which the function $f(x) = \begin{cases} \dfrac{\sin[-x^2]}{[-x^2]}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$ is continuous at $x = 0$ is

(a) $\alpha = 0$ (b) $\alpha = \sin(-1)$

(c) $\alpha = \sin(1)$ (d) $\alpha = 1$

48. A particle starts moving from rest from a fixed point in a fixed direction. The distance s from the fixed point at a time t is given by $s = t^2 + at - b + 17$, where a, b are real numbers. If the particle comes to rest after 5 sec at a distance of $s = 25$ units from the fixed point, then values of a and b are respectively

(a) 10, –33 (b) –10, –33
(c) –8, 33 (d) –10, 33

49. $\lim_{n\to\infty} \frac{\sqrt{1}+\sqrt{2}+...+\sqrt{n-1}}{n\sqrt{n}} =$

(a) 1/2 (b) 1/3 (c) 2/3 (d) 0

50. If $\lim_{x\to 0} \frac{axe^x - b\log(1+x)}{x^2} = 3$, then the values of a and b are respectively

(a) 2, 2 (b) 1, 2 (c) 2, 1 (d) 2, 0

51. Area of the region bounded by $y = |x|$ and $y = -|x| + 2$ is

(a) 4 sq. units (b) 3 sq. units
(c) 2 sq. units (d) 1 sq. unit

52. Let $d(n)$ denote the number of divisors of n including 1 and itself. Then $d(225)$, $d(1125)$ and $d(640)$ are

(a) in A.P. (b) in H.P.
(c) in G.P. (d) consecutive integers

53. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}2a$ has a real solution if

(a) $|a| > \frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}} < |a| < \frac{1}{\sqrt{2}}$
(c) $|a| > \frac{1}{2\sqrt{2}}$ (d) $|a| \le \frac{1}{2\sqrt{2}}$

54. If $2 + i$ and $\sqrt{5} - 2i$ are the roots of the equation $(x^2 + ax + b)(x^2 + cx + d) = 0$, where a, b, c, d are real constants, then product of all roots of the equation is

(a) 40 (b) $9\sqrt{5}$ (c) 45 (d) 35

55. Let $P(x)$ be a polynomial, which when divided by $x - 3$ and $x - 5$ leaves remainders 10 and 6 respectively. If the polynomial is divided by $(x - 3)(x - 5)$ then the remainder is

(a) $-2x + 16$ (b) 16
(c) $2x - 16$ (d) 60

56. The integrating factor of the differential equation $\frac{dy}{dx} + (3x^2 \tan^{-1} y - x^3)(1 + y^2) = 0$ is

(a) e^{x^2} (b) e^{x^3} (c) e^{3x^2} (d) e^{3x^3}

57. If $y = e^{-x}\cos 2x$, then which of the following differential equation is satisfied?

(a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$ (b) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 2y = 0$
(c) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 2y = 0$ (d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 5y = 0$

58. Let $f(x)$ denote the fractional part of a real number x. Then the value of $\int_0^{\sqrt{3}} f(x^2)dx$ is

(a) $2\sqrt{3} - \sqrt{2} - 1$ (b) 0
(c) $\sqrt{2} - \sqrt{3} + 1$ (d) $\sqrt{3} - \sqrt{2} + 1$

59. Let $S = \{(a, b, c) \in N \times N \times N : a + b + c = 21, a \le b \le c\}$ and $T = \{(a, b, c) \in N \times N \times N : a, b, c \text{ are in A.P.}\}$, where N is the set of all natural numbers. Then the number of elements in the set $S \cap T$ is

(a) 6 (b) 7 (c) 13 (d) 14

60. Let $y = e^{x^2}$ and $y = e^{x^2}\sin x$ be two given curves. Then the angle between the tangents to the curves at any point of their intersection is

(a) 0 (b) π (c) $\pi/2$ (d) $\pi/4$

CATEGORY–II (Q61 to Q70)

Each question has one correct option and carries 2 mark, for each wrong answer 1/2 mark will be deducted.

61. A person goes to office by car or scooter or bus or train, probability of which are 1/7, 3/7, 2/7 and 1/7 respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is 2/9, 1/9, 4/9 and 1/9 respectively. Given that he reached office in time, the probability that he travelled by a car is

(a) 1/7 (b) 2/7 (c) 3/7 (d) 4/7

62. The value of $\int \frac{(x-2)dx}{\{(x-2)^2(x+3)^7\}^{1/3}}$ is

(a) $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{4/3} + c$ (b) $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{3/4} + c$
(c) $\frac{5}{12}\left(\frac{x-2}{x+3}\right)^{4/3} + c$ (d) $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{5/3} + c$

63. In a triangle ABC, $\angle C = 90°$, r and R are the inradius and circumradius of the triangle ABC respectively, then $2(r + R)$ is equal to

(a) $b + c$ (b) $c + a$
(c) $a + b$ (d) $a + b + c$

64. Let α, β be two distinct roots of $a\cos\theta + b\sin\theta = c$, where a, b and c are three real constants and $\theta \in [0, 2\pi]$. Then $\alpha + \beta$ is also a root of the same equation if

(a) $a + b = c$ (b) $b + c = a$

(c) $c + a = b$ (d) $c = a$

65. For a matrix, $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$, if U_1, U_2 and U_3 are 3×1 column matrices satisfying

$AU_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $AU_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$, $AU_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and U is a 3×3 matrix whose columns are U_1, U_2 and U_3. Then sum of the elements of U^{-1} is

(a) 6 (b) 0 (c) 1 (d) 2/3

66. Let $f: R \to R$ be differentiable at $x = 0$. If $f(0) = 0$ and $f'(0) = 2$, then the value of

$\lim\limits_{x \to 0} \frac{1}{x}[f(x) + f(2x) + f(3x) + ... + f(2015x)]$ is

(a) 2015 (b) 0

(c) 2015×2016 (d) 2015×2014

67. If x and y are digits such that $17! = 3556xy428096000$, then $x + y$ equals

(a) 15 (b) 6 (c) 12 (d) 13

68. Let $f: N \to R$ be such that $f(1) = 1$ and $f(1) + 2f(2) + 3f(3) + ... + nf(n) = n(n + 1)f(n)$, for all $n \in N$, $n \geq 2$, where N is the set of natural numbers and R is the set of real numbers. Then the value of $f(500)$ is

(a) 1000 (b) 500 (c) 1/500 (d) 1/1000

69. If 5 distinct balls are placed at random into 5 cells, then the probability that exactly one cell remains empty is

(a) 48/125 (b) 12/125 (c) 8/125 (d) 1/125

70. A survey of people in a given region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was 10 times the probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the region is 0.006, what is the probability of death due to lung cancer given that a person is a smoker?

(a) 1/140 (b) 1/70 (c) 3/140 (d) 1/10

CATEGORY–III (Q71 to Q80)

Each question has one or more correct options, choosing which will fetch maximum 2 marks on pro rata basis. However, choice of any wrong option(s) will fetch zero mark for the question.

71. If $\cos x$ and $\sin x$ are solutions of the differential equation $a_0\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$, where a_0, a_1, a_2 are real constants then which of the following is/are always true?

(a) $A\cos x + B\sin x$ is a solution, where A and B are real constants.

(b) $A\cos\left(x + \frac{\pi}{4}\right)$ is a solution, where A is a a real constant.

(c) $A\cos x \sin x$ is a solution, where A is real constant.

(d) $A\cos\left(x + \frac{\pi}{4}\right) + B\sin\left(x - \frac{\pi}{4}\right)$ is a solution, where A and B are real constants.

72. Which of the following statements is/are correct for $0 < \theta < \frac{\pi}{2}$?

(a) $(\cos\theta)^{1/2} \leq \cos\frac{\theta}{2}$ (b) $(\cos\theta)^{3/4} \geq \cos\frac{3\theta}{4}$

(c) $\cos\frac{5\theta}{6} \geq (\cos\theta)^{5/6}$ (d) $\cos\frac{7\theta}{8} \leq (\cos\theta)^{7/8}$

73. Which of the following is/are always false?

(a) A quadratic equation with rational coefficients has zero or two irrational roots.

(b) A quadratic equation with real coefficients has zero or two non-real roots.

(c) A quadratic equation with irrational coefficients has zero or two rational roots.

(d) A quadratic equation with integer coefficients has zero or two irrational roots.

74. If the straight line $(a - 1)x - by + 4 = 0$ is normal to the hyperbola $xy = 1$ then which of the following does not hold?

(a) $a > 1, b > 0$ (b) $a > 1, b < 0$

(c) $a < 1, b < 0$ (d) $a < 1, b > 0$

75. Let f be any continuously differentiable function on $[a, b]$ and twice differentiable on (a, b) such that

$f(a) = f'(a) = 0$ and $f(b) = 0$. Then
(a) $f''(a) = 0$
(b) $f'(x) = 0$ for some $x \in (a, b)$
(c) $f''(x) = 0$ for some $x \in (a, b)$
(d) $f'''(x) = 0$ for some $x \in (a, b)$

76. A relation ρ on the set of real number R is defined as follows: "$x \rho y$ if and only if $xy > 0$". Then which of the following is/are true?
(a) ρ is reflexive and symmetric.
(b) ρ is symmetric but not reflexive
(c) ρ is symmetric and transitive
(d) ρ is an equivalence relation

77. Suppose a machine produces metal parts that contains some defective parts with probability 0.05. How many parts should be produced in order that the probability of at least one part being defective is 1/2 or more? (Given $\log_{10}95 = 1.977$ and $\log_{10}2 = 0.3$)
(a) 11 (b) 12 (c) 15 (d) 14

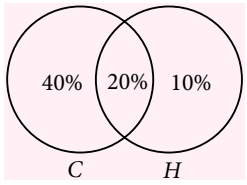
78. Let $f : R \to R$ be such that $f(2x - 1) = f(x)$ for all $x \in R$. If f is continuous at $x = 1$ and $f(1) = 1$, then
(a) $f(2) = 1$ (b) $f(2) = 2$
(c) f is continuous only at $x = 1$
(d) f is continuous at all points

79. Let $16x^2 - 3y^2 - 32x - 12y = 44$ represents a hyperbola. Then
(a) length of the transverse axis is $2\sqrt{3}$
(b) length of each latus rectum is $32/\sqrt{3}$
(c) eccentricity is $\sqrt{19/3}$
(d) equation of a directrix is $x = \frac{\sqrt{19}}{3}$

80. For the function $f(x) = \left[\frac{1}{[x]}\right]$, where $[x]$ denotes the greatest integer less than or equal to x, which of the following statements are true?
(a) The domain is $(-\infty, \infty)$.
(b) The range is $\{0\} \cup \{-1\} \cup \{1\}$.
(c) The domain is $(-\infty, 0) \cup [1, \infty)$.
(d) The range is $\{0\} \cup \{1\}$

SOLUTIONS

1. (a) :



$\therefore$ Only car or house = 40% + 10% = 50%
$\therefore$ Probability = 1/2

2. (c) : Alphabetically : C, C, H, I, N, O
Permutation : CC * * * * $\to \underline{|4}$
CH * * * * $\to \underline{|4}$
CI * * * * $\to \underline{|4}$
CN * * * * $\to \underline{|4}$
Next $\to$ COCHIN
$\therefore$ No. of words before COCHIN = $\underline{|4} \times 4 = 96$

3. (c) : $f(x) = \int_0^x f(t)dt \Rightarrow f'(x) = f(x)$
$\Rightarrow f(x) = e^x$
$\therefore f(\log_e 5) = e^{\log_e 5} = 5$

4. (b) : $\lim_{x\to 2} \frac{\int_2^x 3t^2 dt}{x-2}$ $\left[\frac{0}{0}\text{ form}\right]$

$= \lim_{x\to 2} \frac{\frac{d}{dx}\int_2^x 3t^2 dt}{\frac{d}{dx}(x-2)}$ [using L' Hospital rule]

$= \lim_{x\to 2} \frac{3x^2}{1} = 12$

5. (b) : $\frac{\cos\frac{2x}{3}\cos\frac{x}{3} + \sin\frac{2x}{3}\sin\frac{x}{3}}{\sin\frac{2x}{3}\cos\frac{x}{3}} = \text{cosec}\,\frac{kx}{3}$

$\Rightarrow \frac{\cos\left(\frac{2x}{3} - \frac{x}{3}\right)\cdot \text{cosec}\,\frac{2x}{3}}{\cos\frac{x}{3}} = \text{cosec}\,\frac{kx}{3} \Rightarrow k = 2$

6. (b) :

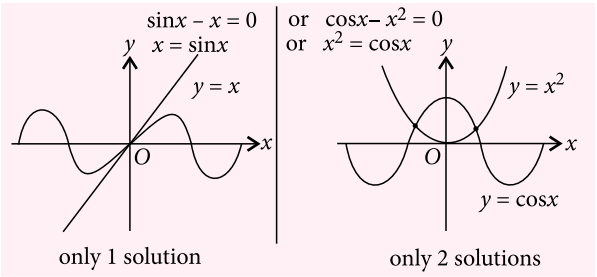
$\sqrt{4\cos^4\theta + 4\sin^2\theta\cos^2\theta} + 2\cot\theta \cdot 2\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

$= \sqrt{4\cos^2\theta(\cos^2\theta + \sin^2\theta)} + 2\cot\theta\left\{1 + \cos\left(\frac{\pi}{2} - \theta\right)\right\}$

$= 2|\cos\theta| + 2\cot\theta\,(1 + \sin\theta) = -2\cos\theta + 2\cot\theta + 2\cos\theta$

$= 2\cot\theta \quad \left[\because \theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \therefore \cos\theta < 0\right]$

7. (c) :



$\therefore$ Total 3 solutions exist.

8. (d) : Augmented matrix is

$$\begin{bmatrix} 1 & -2 & 1 & -4 \\ 2 & -1 & -2 & 2 \\ 1 & 1 & \lambda & 4 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$, we get

$$\begin{bmatrix} 1 & -2 & 1 & -4 \\ 0 & 3 & -4 & 10 \\ 0 & 3 & \lambda-1 & 8 \end{bmatrix}$$

Applying $R_3 \to R_3 - R_2$,we get

$$\begin{bmatrix} 1 & -2 & 1 & -4 \\ 0 & 3 & -4 & 10 \\ 0 & 0 & \lambda+3 & -2 \end{bmatrix}$$

$\therefore$ System has no solution if $\lambda + 3 = 0 \Rightarrow \lambda = -3$

9. (a) :

$$f(x) = \begin{vmatrix} 1 & x & 0 \\ 2x & x(x-1) & 0 \\ 3x(x-1) & x(x-1)(x-2) & 0 \end{vmatrix}$$

Applying $C_3 \to C_3 - C_1 - C_2$, we get

$f(x) = 0 \Rightarrow f(100) = 0$

10. (b) : $\because \quad |\vec{a}+\lambda\vec{b}|^2 = (\vec{a}+\lambda b)\cdot(\vec{a}+\lambda\vec{b})$

$= |\vec{a}|^2 + \lambda\vec{b}\cdot\vec{a} + \lambda\vec{a}\cdot\vec{b} + \lambda^2|\vec{b}|^2$

$= |\vec{a}|^2 + 0 + 0 + \lambda^2|\vec{b}|^2 \quad [\vec{a}\perp\vec{b}]$

$= |\vec{a}|^2 + \lambda^2|\vec{b}|^2 \geq |\vec{a}|^2,$

$\therefore \quad |\vec{a}+\lambda\vec{b}| \geq |\vec{a}|$ if $\vec{a}\perp\vec{b}$

11. (a) : Let $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be the given vectors.

$\vec{a}-\vec{b} = -3\hat{i}, \vec{b}-\vec{c} = \hat{i}+2\hat{k}, \vec{c}-\vec{d} = (1-\lambda)\hat{j}-2\hat{k}$

For coplanarity, we have

$$\begin{vmatrix} -3 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1-\lambda & -2 \end{vmatrix} = 0 \Rightarrow -3\begin{vmatrix} 0 & 2 \\ 1-\lambda & -2 \end{vmatrix} = 0$$

$\Rightarrow \lambda = 1$

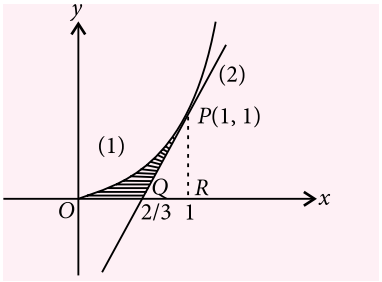
12. (a) : $\sin^{-1}\left\{\dfrac{x}{1-\left(\dfrac{-x}{2}\right)}\right\} = \dfrac{\pi}{6} \Rightarrow \dfrac{2x}{2+x} = \dfrac{1}{2} \Rightarrow x = \dfrac{2}{3}$

13. (a) : Let the equation of the curve be $y = x^3$... (1)

$\therefore \quad \left[\dfrac{dy}{dx}\right]_{(1,1)} = 3$

$PQ : y - 1 = 3(x - 1) \Rightarrow y = 3x - 2$ ... (2)

Required area $OQPO = \int_0^1 x^3 dx - \int_{2/3}^1 (3x-2)dx = \dfrac{1}{12}$



14. (c) : $\log_{0.2}(x-1) > \log_{0.04}(x+5)$

$\Rightarrow \log_{0.2}(x-1) > \dfrac{1}{2}\log_{0.2}(x+5)$

$\Rightarrow (x-1)^2 < x+5$ [$\because$ base < 1]

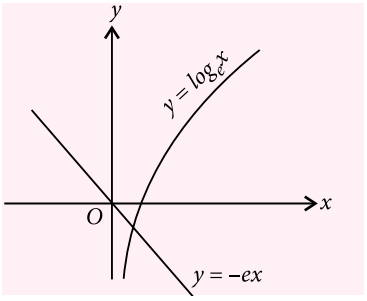
$\Rightarrow x^2 - 3x - 4 < 0$

$\Rightarrow (x-4)(x+1) < 0 \Rightarrow -1 < x < 4$

But $\log_{0.2}(x-1)$ is defined if $x - 1 > 0$ *i.e.* $x > 1$

$\therefore \quad 1 < x < 4$

15. (b) :



$y = \log_e x$ ... (1)

$y = -ex$ ... (2)

From graph it is clear that (1) and (2) meet at one point.

$\Rightarrow$ No. of real root of $\log_e x + ex = 0$ is 1.

16. (a) : Let $f(x) = a_0 x + a_1\dfrac{x^2}{2} + a_2\dfrac{x^3}{3} + a_3\dfrac{x^4}{4}$

$f'(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$\because f(0) = 0 = f(1), \left[\because f(1) = a_0 + \dfrac{a_1}{2} + \dfrac{a_2}{3} + \dfrac{a_3}{4} = 0\right]$

$\therefore$ $f(x)$ satisfies all conditions of Rolle's theorem.

$f'(c) = 0$ for some $c \in (0, 1)$

$\therefore$ $f'(x) = 0$ has a real root in $[0, 1]$

17. (d) : $f(x)$ is continuous at $x = a$ if $\lim\limits_{h\to 0} f(a-h) = f(a)$

we consider the continuity at $x = k\pi$ $(k \in I)$

$\therefore \quad \lim\limits_{h\to 0} f(k\pi + h) = f(k\pi)$ for continuity at $x = k\pi\,(k \in I)$ ...(1)

Now, $\lim_{h\to 0} f(k\pi+h)$

$$=\begin{cases}\lim_{h\to 0} 0 & \text{if } k\pi+h \text{ is rational}\\ \lim_{h\to 0}\sin|k\pi+h|, & \text{if } k\pi+h \text{ is irrational}\end{cases}$$

$= 0$ [$\because \sin|k\pi| = 0$ as $k \in I$]

And $f(k\pi) = 0$ [$\because k\pi$ is irrational as $k \in I$] $\therefore$ (1) is true

18. (b) : $y^2 - 4y = 4x - 4a$

$\Rightarrow (y-2)^2 = 4(x - a + 1)$

$\Rightarrow$ vertex is $(a-1, 2)$

$\because$ $(a-1, 2)$ lies between parallel lines $x + y = 3$ and $2x + 2y - 1 = 0$

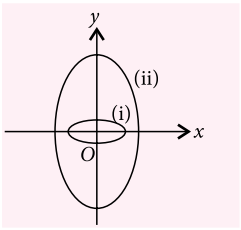
$\therefore$ $\{(a-1) + 2 - 3\}$ and $\{2(a-1) + 2(2) - 1\}$ will be of opposite sign.

i.e. $(a-2)$ and $(2a+1)$ will be of opposite sign.

$\Rightarrow (a-2)(2a+1) < 0 \Rightarrow -\frac{1}{2} < a < 2$

19. (d) : $4x^2+9y^2=1 \Rightarrow \frac{x^2}{\left(\frac{1}{2}\right)^2}+\frac{y^2}{\left(\frac{1}{3}\right)^2}=1$ ... (i)

and $4x^2+y^2=4 \Rightarrow \frac{x^2}{1^2}+\frac{y^2}{2^2}=1$ ...(ii)



From graph, it is seen that there is no solution of (i) and (ii).

20. (c) : If $\frac{x-\lambda}{3}=\frac{y-1}{2+\lambda}=\frac{z-3}{-1}=K$ (say) ...(i)

lie on plane $x - 2y = 0$ ...(ii)

then $3\cdot(1) + (2+\lambda)\cdot(-2) + (-1).0 = 0$

$\Rightarrow 3 - 4 - 2\lambda = 0 \Rightarrow \lambda = -\frac{1}{2}$

21. (c) :

$$f(\theta)=\begin{vmatrix}1 & \tan\theta & 1\\ -\tan\theta & 1 & \tan\theta\\ -1 & -\tan\theta & 1\end{vmatrix}$$

$$=\begin{vmatrix}1 & \tan\theta & 1\\ 0 & 1+\tan^2\theta & 2\tan\theta\\ 0 & 0 & 2\end{vmatrix}\begin{bmatrix}R_2\to R_2+R_1\tan\theta\\ R_3\to R_3+R_1\end{bmatrix}$$

$$=2\begin{vmatrix}1 & \tan\theta\\ 0 & \sec^2\theta\end{vmatrix}=2\sec^2\theta\in[2,\infty) \text{ as } 0\le\theta<\frac{\pi}{2}$$

22. (c) : $\because$ $AB = B$ ...(i) and $BA = A$ ...(ii)

$\therefore$ $A^2 + B^2 = A\cdot A + B\cdot B$

$= A(BA) + B(AB)$ [using (i) and (ii)]

$= (AB)A + (BA)B$ [Associative law]

$= BA + AB$ [using (i) and (ii)]

$= A + B$

23. (b) : $\begin{vmatrix}-\omega^2 & \omega^2 & -\omega\\ -\omega & \omega & -\omega^2\\ -1 & \omega & -\omega^2\end{vmatrix}$ [$\because 1+\omega+\omega^2=0$]

$$=\begin{vmatrix}0 & \omega^2 & -\omega\\ 0 & \omega & -\omega^2\\ (-1+\omega) & \omega & -\omega^2\end{vmatrix} \quad (C_1\to C_1+C_2)$$

$$=(-1+\omega)\begin{vmatrix}\omega^2 & -\omega\\ \omega & -\omega^2\end{vmatrix}=(\omega-1)(-\omega^4+\omega^2)$$

$$=(\omega-1)(-\omega+\omega^2)$$

$= -\omega^2 + \omega + \omega^3 - \omega^2 = -2\omega^2 + \omega + 1$

$= -3\omega^2 + (\omega^2 + \omega + 1) = -3\omega^2$

24. (d) : $2\tan^{-1}2-\tan^{-1}\frac{3}{4}=\pi+\tan^{-1}\frac{2(2)}{1-2^2}-\tan^{-1}\frac{3}{4}$

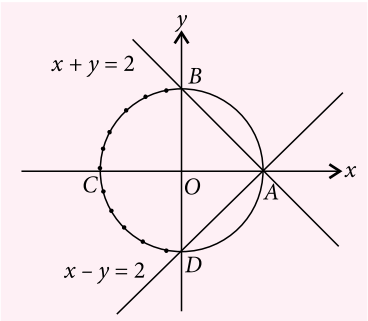
$$=\pi+\tan^{-1}\left(-\frac{4}{3}\right)-\tan^{-1}\frac{3}{4}$$

$$=\pi-\tan^{-1}\frac{4}{3}-\tan^{-1}\frac{3}{4}=\pi-\left(\tan^{-1}\frac{4}{3}+\cot^{-1}\frac{4}{3}\right)$$

$$=\frac{\pi}{2}\left(\because \tan^{-1}\theta+\cot^{-1}\theta=\frac{\pi}{2}\right)$$

25. (c): $\because (2\cos\theta, 2\sin\theta)$ is a point on circle $x^2 + y^2 = 4$ and the region between $x + y = 2$ and $x - y = 2$ containing origin is $ABCDA$.

$\therefore$ All pts will be scattered along BCD on circle for which θ will belong to $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$



26. (c) : We must have

$$\frac{|x-2y+1|}{\sqrt{1^2+(-2)^2}}=\sqrt{5} \text{ and } \frac{|2x+3y-1|}{\sqrt{2^2+3^2}}=\sqrt{13}$$

$\Rightarrow$ $x - 2y + 1 = \pm 5$ and $2x + 3y - 1 = \pm 13$

On solving, we will get exactly 4 points

27. (d) : $a^n + b^n = c^n + d^n$ ... (1)

Putting $n = 1$, $a + b = c + d$ ... (2)

Putting $n = 2$, $a^2 + b^2 = c^2 + d^2$ ... (3)

From (2), $a^2 + b^2 + 2ab = c^2 + d^2 + 2cd$

$\Rightarrow$ $ab = cd$ ... (4) [From (3)]

Now, from (2),

$a^3 + b^3 + 3ab(a + b) = c^3 + d^3 + 3cd(c + d)$

$\Rightarrow$ $a^3 + b^3 + 3ab(a + b) = c^3 + d^3 + 3ab(a + b)$ [Using (2) and (4)]

$\therefore$ $a^3 + b^3 = c^3 + d^3$ $\Rightarrow$ (1) is true for $n = 3$

From (3), $a^2 - d^2 = c^2 - b^2$

$\Rightarrow$ $(a + d)(a - d) = (c + b)(c - b)$

$\Rightarrow$ $a + d = c + b$ [Using (2)]

$\Rightarrow$ $a - b = c - d$ ... (5)

On multiplying (2) and (5), $a^2 - b^2 = c^2 - d^2$

28. (a) : $\because$ $x^2 - px + 1 = 0$ changes to $x^2 + px + 1 = 0$ if we put $x = -x$

$\therefore$ If α, β be the roots of $x^2 - px + 1 = 0$ then $-\alpha, -\beta$ will be the roots of $x^2 - px + 1 = 0$

$\Rightarrow$ $\gamma = -\alpha$ or $-\beta$

$\Rightarrow$ $\alpha + \gamma = 0$ or $\beta + \gamma = 0$

$\Rightarrow$ $(\alpha + \gamma)(\beta + \gamma) = 0$

29. (None of the options is correct) :

$$T_{r+1} = {}^{100}C_r(3^{1/5})^{100-r}(7^{1/3})^r$$

$$= {}^{100}C_r \cdot 3^{\frac{100-r}{5}} \cdot 7^{\frac{r}{3}}$$

No. of rational terms will be obtained if $r = 0, 15, 30, 45, 60, 75$ or 90

$\therefore$ No. of irrational terms = total terms $- 7$

$= 101 - 7 = 94$

30. (c) : $4x^2 + (4 - p)x + (q + 1) \neq 0$

$\Rightarrow$ no real root of $4x^2 + (4 - p)x + (q + 1) = 0$

$\Rightarrow$ $D < 0$ $\Rightarrow$ $(4 - p)^2 - 4 \cdot 4(q + 1) < 0$

$\Rightarrow$ $p^2 - 8p - 16q < 0$

31. (a) : If $x > 0$, $x+\frac{4}{x} \geq 4$ [using A.M. $\geq$ G.M]

If $x < 0$, $x+\frac{4}{x} \leq -4$

Now, $f(x)=\frac{x^2-x+4}{x^2+x+4}=\frac{\left(x+\frac{4}{x}\right)-1}{\left(x+\frac{4}{x}\right)+1}$

Putting $x = 4, f(x) = \frac{3}{5}$

Putting $x = -4, f(x) = \frac{5}{3}$

$\therefore$ $f(x) \in \left[\frac{3}{5}, \frac{5}{3}\right]$

32. (None of the options is correct) :

Given expression

$$=\frac{1}{2}[4x^2+2y^2+4xy+4x-6y+16]$$

$$=\frac{1}{2}[(2x)^2+y^2+2(2x\cdot y+y\cdot 1+1\cdot 2x)+(y^2-8y+16)]$$

$$=\frac{1}{2}[(2x+y+1)^2-1+(y-4)^2] \geq -\frac{1}{2}$$

Equality sign will hold only if $2x + y + 1 = 0$ & $y - 4 = 0$

i.e. if $y = 4$, $x=\frac{-5}{2}$ $\therefore$ Least value $=-\frac{1}{2}$

33. (d) : $\because$ $f : [-2, 2] \to R$ is continuous function and assumes only irrational values

$\therefore$ $f(x) = k$ (some irrational no.)

However, $f(\sqrt{2})=\sqrt{2}$ (given) $\Rightarrow$ $k=\sqrt{2}$

$\therefore$ $f(x)=\sqrt{2} \Rightarrow f(\sqrt{2}-1)=\sqrt{2}$

34. (a) : The minimum value will be obtained when $\sin\theta = \cos\theta$ *i.e.* $\theta=\frac{\pi}{4} \text{in}\left(0,\frac{\pi}{2}\right)$

$\therefore$ Min value $=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{2}{1}=\sqrt{2}+2$

35. (b) : Let $\omega=\frac{-1+\sqrt{3}i}{2}$ and $\omega^2=\frac{-1-\sqrt{3}i}{2}$

$\therefore$ Given expression $=\left(\frac{-2\omega^2}{-2\omega}\right)^{64}+\left(\frac{-2\omega}{-2\omega^2}\right)^{64}$

$=\omega^{64}+\frac{1}{\omega^{64}}=\omega+\frac{1}{\omega}$ $[\because \omega^3 = 1]$

$= \omega + \omega^2 = -1$

36. (b) : $2=\left|z-\frac{3}{z}\right| \geq |z|-\left|\frac{3}{z}\right| \Rightarrow 2 \geq |z|-\frac{3}{|z|}$

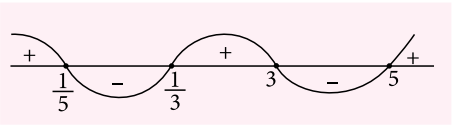
$\Rightarrow$ $|z|^2 - 2|z| - 3 \leq 0$ $\Rightarrow$ $(|z| - 1)^2 \leq 4$

$\Rightarrow$ $||z| - 1| \leq 2$ $\Rightarrow$ $-2 \leq |z| - 1 \leq 2$

$\Rightarrow \ -1 \le |z| \le 3 \ \Rightarrow \ |z| \le 3$

$\therefore$ Maximum value of $|z| = 3$

37. (d) : $\dfrac{(x-5)(5x-1)}{(3x-1)(x-3)} < 0 \ \Rightarrow \ \dfrac{\left(x-\frac{1}{5}\right)(x-5)}{\left(x-\frac{1}{3}\right)(x-3)} < 0$



$\therefore \ \dfrac{1}{5} < x < \dfrac{1}{3}$ or $3 < x < 5$

38. (d)

39. (a) : Variance of first 20 natural nos. is given by

$$6^2 = \frac{\sum n^2}{n} - \left(\frac{\sum n}{n}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = \frac{(n+1)}{12}(4n+2-3n-3)$$

$$= \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12} = \frac{20^2-1}{12} \ [\because \ n = 20]$$

$$= \frac{399}{12} = \frac{133}{4}$$

40. (b): Here, p = getting a head $= \dfrac{1}{2} \ \therefore \ q = 1 - p = \dfrac{1}{2}$

$\because \ P(x = 3) = P(x = 5)$ (given)

$\therefore \ {}^nC_3 \ q^{n-3}p^3 = {}^nC_5 q^{n-5}p^5 \Rightarrow {}^nC_3 = {}^nC_5 \left[\because p = q = \dfrac{1}{2}\right]$

$\Rightarrow \ n = 3 + 5 = 8$

$\therefore \ P(x=1) = {}^8C_1 \, q^{8-1}p^1 = 8 \cdot \left(\dfrac{1}{2}\right)^8 = \dfrac{1}{32}$

41. (a) : Without any restriction total number of ways of arranging the letters $= \dfrac{\lfloor 11}{\lfloor 2 \lfloor 2} = n(S)$

Considering 2 B's as a single letter then $\dfrac{\lfloor 10}{\lfloor 2} = n(E)$

$\therefore \ P(E) = \dfrac{n(E)}{n(S)} = \dfrac{\lfloor 10}{\lfloor 2} \times \dfrac{\lfloor 2 \lfloor 2}{\lfloor 11} = \dfrac{2}{11}$

42. (d) : We have $t = x - \alpha$, $y = -16$, $-16 = \alpha x$

$$\therefore \ t = -\frac{16}{\alpha} - \alpha = -\left(\alpha + \frac{16}{\alpha}\right) \qquad \ldots(i)$$

$$= z + \frac{16}{z} \ [\text{Let } \alpha = -z]$$

$$= 2\left(\frac{z + \frac{16}{z}}{2}\right) \ge 2\sqrt{z \cdot \frac{16}{z}} \quad \therefore \ t \ge 8$$

43. (b) : $\cos^2 A = \dfrac{b^2 + c^2}{a^2}$

But, $\cos^2 A < 1$ as in a triangle ABC

$\cos A \ne 1$

$$\therefore \ \frac{b^2 + c^2}{a^2} < 1 \ \Rightarrow \ b^2 + c^2 - a^2 < 0 \Rightarrow \cos A < 0$$

$$\Rightarrow \ \frac{\pi}{2} < A < \pi$$

44. (a) : $\because \ |\cos x| \ge \sin x$ is true in $\left[0, \dfrac{3\pi}{2}\right]$ only if $x \in \left[0, \dfrac{\pi}{4}\right]$ and $\left[\dfrac{3\pi}{4}, \dfrac{3\pi}{2}\right]$

$\therefore$ Required answer $= \left[0, \dfrac{\pi}{4}\right] \cup \left[\dfrac{3\pi}{4}, \dfrac{3\pi}{2}\right]$

45. (c) : $\begin{vmatrix} \sin x + 2\cos x & 0 & \cos x \\ \sin x + 2\cos x & \sin x - \cos x & \cos x \\ \sin x + 2\cos x & \cos x - \sin x & \sin x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + (C_2 + C_3)$; $C_2 \to C_2 - C_3$

$$\Rightarrow \ (\sin x + 2\cos x)(\sin x - \cos x)\begin{vmatrix} 1 & 0 & \cos x \\ 1 & 1 & \cos x \\ 1 & -1 & \sin x \end{vmatrix} = 0$$

$$\Rightarrow \ (\sin x + 2\cos x)(\sin x - \cos x)\begin{vmatrix} 1 & 0 & \cos x \\ 0 & 1 & 0 \\ 0 & -1 & \sin x - \cos x \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$, we get

$\Rightarrow \ \sin x + 2\cos x = 0$ or $\sin x - \cos x = 0$

$\Rightarrow \ \tan x = -2$ or $\tan x = 1$

$\Rightarrow \ x$ has only one solution in $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

46. (b) : $\because$ Exactly one of $x_1, x_2, \ldots, x_{15}$ will be 1 and so exactly any one factor of $(x_1 - 1), (x_2 - 1), (x_3 - 1), \ldots$ or $(x_{15} - 1)$ will be zero.

$\therefore \ (x_1 - 1)(x_2 - 1)(x_3 - 1) \ldots (x_{15} - 1) = 0$

47. (c) : $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \dfrac{\sin[-x^2]}{[-x^2]}$

$$= \lim_{x \to 0} \frac{\sin(-1)}{(-1)} = \sin 1 \quad \text{and } f(0) = \alpha$$

$\therefore \ f(x)$ will be continuous at $x = 0$ if $\alpha = \sin 1$.

48. (b) : $s = t^2 + at - b + 17$ ...(i)

$$\frac{ds}{dt} = v = 2t + a \qquad \ldots(ii)$$

$\because$ Particle comes to rest after 5 sec. at a distance of 25 units

$\therefore$ When $t = 5, s = 25, v = 0$

$\therefore$ From (ii) $0 = 10 + a \Rightarrow a = -10$

From (i), $25 = 25 + (-10)5 - b + 17 \Rightarrow b = -33$

49. (c) :

$$\lim_{n\to\infty}\frac{1}{n}\left[\sqrt{\frac{1}{n}}+\sqrt{\frac{2}{n}}+....+\sqrt{\frac{n-1}{n}}\right]=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n-1}\sqrt{\frac{r}{n}}$$

$$=\int_0^1\sqrt{x}\,dx=\left[\frac{2}{3}x^{3/2}\right]_0^1=\frac{2}{3}$$

50. (a) :

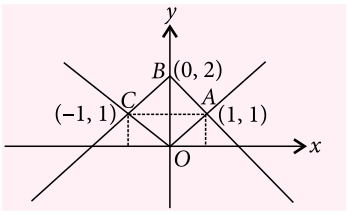
$$\lim_{x\to0}\frac{ax\left(1+\frac{x}{\lfloor1}+\frac{x^2}{\lfloor2}+.....\right)-b\left(x-\frac{x^2}{2}+\frac{x^3}{3}-....\right)}{x^2}=3$$

As limit exists, we must have,

$$a-b=0 \text{ and } \frac{a}{\lfloor1}+\frac{b}{2}=3$$

$\Rightarrow a = b = 2$

51. (c) :



Here, $OABC$ is the bounded region.

Area of right angled $\Delta OAB = \frac{1}{2}\times\sqrt{2}\times\sqrt{2} = 1$ sq. unit

$\therefore$ Area of $OABC = 2 \times 1 = 2$ sq. units

52. (c) : $225 = 3^2\cdot5^2 \Rightarrow d(225) = (2 + 1)(2 + 1) = 9 = 1$

$1125 = 3^2\cdot5^3 \Rightarrow d(1125) = (2 + 1)(3 + 1) = 12$

$640 = 2^7\cdot5^1 \Rightarrow d(640) = (7 + 1)(1 + 1) = 16$

$\because 12^2 = 9 \times 16 \Rightarrow d(225), d(1125)$ and $d(640)$ are in G.P.

53. (d) : $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$ for all $x\in[-1,1]$

$$\Rightarrow -\frac{\pi}{4}\le\sin^{-1}2a\le\frac{\pi}{4}\Rightarrow-\frac{1}{\sqrt{2}}\le2a\le\frac{1}{\sqrt{2}}$$

$$\Rightarrow -\frac{1}{2\sqrt{2}}\le a\le\frac{1}{2\sqrt{2}}\Rightarrow|a|\le\frac{1}{2\sqrt{2}}$$

54. (d) : $\because a, b, c, d$ are real.

$\therefore$ If $2 + i$ and $\sqrt{5}-2i$ be roots then other roots will be their conjugate.

$\therefore$ Product of all roots

$= (2 + i)(2 - i)(\sqrt{5}-2i)(\sqrt{5}+2i)$

$= (4 - i^2)(5 - 4i^2) = 45$

55. (a) : $\because$ $P(x)$ be such a polynomial which when divided by $x - 3$ and $x - 5$ (both linear) leaves the remainder 10 and 6 respectively. (both constant)

$\therefore$ When $P(x)$ will be divided by $(x - 3)(x - 5)$, the remainder will be linear, let $ax + b$

$\therefore$ Accordingly, $P(3) = 10$ and $P(5) = 6$ gives

$3a + b = 10$ and $5a + b = 6$

On solving, $a = -2, b = 16$

$\Rightarrow$ Remainder is $-2x + 16$

56. (b) : We can write, $\frac{1}{1+y^2}\cdot\frac{dy}{dx}+3x^2\tan^{-1}y=x^3$

On putting $\tan^{-1}y = t$, we get $\frac{1}{1+y^2}\cdot\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore$ Given equation becomes $\frac{dt}{dx}+3x^2t=x^3$

Here, $P = 3x^2, Q = x^3$

$\therefore$ I.F. $= e^{\int P.dx} = e^{x^3}$

57. (a) : $y = e^{-x}\cos2x$ ...(i)

$\therefore$ $y_1 = e^{-x}(-2\sin2x) - e^{-x}\cos2x$

$= -2e^{-x}\sin2x - y$ ...(ii)

$\Rightarrow y_2 = -2\{e^{-x}(2\cos2x) - e^{-x}\sin2x\} - y_1$

$= -4y + (-y_1 - y) - y_1$ [from (i) and (ii)]

$\therefore$ $y_2 + 2y_1 + 5y = 0$

58. (c) :

$$\int_0^{\sqrt{3}}f(x^2)dx=\int_0^1f(x^2)dx+\int_1^{\sqrt{2}}f(x^2)dx+\int_{\sqrt{2}}^{\sqrt{3}}f(x^2)dx$$

$$=\int_0^1x^2dx+\int_1^{\sqrt{2}}(x^2-1)dx+\int_{\sqrt{2}}^{\sqrt{3}}(x^2-2)dx$$

$$=\left[\frac{x^3}{3}\right]_0^1+\left[\frac{x^3}{3}-x\right]_1^{\sqrt{2}}+\left[\frac{x^3}{3}-2x\right]_{\sqrt{2}}^{\sqrt{3}}$$

$$=\frac{1}{3}+\left(\frac{2\sqrt{2}}{3}-\sqrt{2}\right)-\left(\frac{1}{3}-1\right)+(\sqrt{3}-2\sqrt{3})-\left(\frac{2\sqrt{2}}{3}-2\sqrt{2}\right)$$

$$=\sqrt{2}-\sqrt{3}+1$$

59. (b) : For S, $a + b + c = 21$ ...(i)

For T, $a + c = 2b$ ...(ii)

If considering $S \cap T$, then on solving (i) and (ii), we get $b = 7$ and $a + c = 14$

$\because$ $a\le b\le c$ $\therefore$ $a = 1, 2, 3, 4, 5, 6, 7$

a	b	c
1	7	13
2	7	12
3	7	11
4	7	10
5	7	9
6	7	8
7	7	7

No. of elements in $S \cap T = 7$

60. (a) : $y = e^{x^2}$ ...(i) $\quad y = e^{x^2}\sin x$ ...(ii)

On solving,

$e^{x^2} = e^{x^2}\sin x \Rightarrow \sin x = 1 \Rightarrow x = \frac{\pi}{2} \quad [\because e^{x^2} \neq 0]$

From (i), $\frac{dy}{dx} = e^{x^2}\cdot 2x$ ...(iii)

From (ii), $\frac{dy}{dx} = e^{x^2}\cos x + xe^{x^2}\sin x \quad [\because \cos x = 0]$

For $\sin x = 1$, $\frac{dy}{dx} = 2xe^{x^2} \quad [\because \cos x = 0]$ ...(iv)

$\therefore$ At point of intersection (where $\sin x = 1$), both slopes (of tangents) are same $\Rightarrow$ Required angle = 0

61. (a) : Using Baye's theorem,

$$P(C/T) = \frac{P(T/C)P(C)}{\sum P(T/C)\,P(C)}$$

$$= \frac{\left(1-\frac{2}{4}\right)\frac{1}{7}}{\left(1-\frac{2}{9}\right)\frac{1}{7}+\left(1-\frac{1}{9}\right)\frac{3}{7}+\left(1-\frac{4}{9}\right)\frac{2}{7}+\left(1-\frac{1}{9}\right)\frac{1}{7}} = \frac{1}{7}$$

62. (a) : $\int\left(\frac{x-2}{x+3}\right)^{1/3}\cdot\frac{1}{(x+3)^2}dx$

$$\left[\text{Put } \frac{x-2}{x+3} = z \Rightarrow \frac{5}{(x+3)^2}dx = dz\right]$$

$$= \frac{1}{5}\int z^{1/3}\cdot dz = \frac{1}{5}\cdot\frac{3}{4}\cdot z^{4/3} + c = \frac{3}{20}\left(\frac{x-2}{x+3}\right)^{4/3} + c$$

63. (c) : $\because \quad r = (s-c)\tan\frac{C}{2}$

$\therefore \quad 2r = 2(s-c)\tan 45° = 2s - 2c$

$= a + b + c - 2c = a + b - c$

$= a + b - 2R.$

$\therefore \quad 2(r + R) = a + b$

and $\frac{c}{\sin C} = 2R \Rightarrow \frac{c}{\sin 90°} = 2R. \quad \therefore \quad c = 2R$

64. (d) : $\because$ α, β are roots of $a\cos\theta + b\sin\theta = c$ ... (i)

$\therefore \quad a\cos\alpha + b\sin\alpha = c$ ... (ii)

$a\cos\beta + b\sin\beta = c$ ... (iii)

(iii) – (ii) :

$$a\cdot 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) - b\cdot 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\Rightarrow \tan\left(\frac{\alpha+\beta}{2}\right) = \frac{b}{a} \quad \text{... (iv)}$$

If $\alpha + \beta$ is also a root of (i) then

$a\cos(\alpha + \beta) + b\sin(\alpha + \beta) = c$

$$\Rightarrow \frac{a\left\{1-\left(\frac{b}{a}\right)^2\right\}}{1+\left(\frac{b}{a}\right)^2} + \frac{b\cdot 2\left(\frac{b}{a}\right)}{1+\left(\frac{b}{a}\right)^2} = c \quad [\text{using (iv)}]$$

$$\Rightarrow \frac{a(a^2-b^2)+2b^2a}{a^2+b^2} = c \Rightarrow \frac{a(a^2-b^2+2b^2)}{a^2+b^2} = c$$

$\Rightarrow \quad a = c$

65. (b) : Let $U_1 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, $U_2 = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$ and $U_3 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\because \quad AU_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ 2a+b \\ 3a+2b+c \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow \quad a = 1, b = -2, c = 1$

$$\because \quad AU_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} p \\ 2p+q \\ 3p+2q+r \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$\Rightarrow \quad p = 2, q = -1, r = -4$

$$\because \quad AU_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ 2x+y \\ 3x+2y+z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$\Rightarrow \quad x = 2, y = -1, z = -3$

$$\therefore \quad U = \begin{pmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{pmatrix}. \quad \therefore \quad \text{Adj}\,U = \begin{pmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{pmatrix}$$

$$\therefore \quad U^{-1} = \frac{1}{|U|}(\text{adj}\,U)$$

$\therefore$ Sum of all elements in U^{-1}

$$= \frac{1}{|U|}(-1-2+0-7-5-3+9+6+3) = 0$$

66. (c) :

$$\lim_{x\to 0}\frac{1}{x}[f(x)+f(2x)+f(3x)+...+f(2015x)] \quad \left(\frac{0}{0}\text{ form}\right)$$

$$=\lim_{x\to 0}\frac{[f'(x)+f'(2x)\cdot 2+f'(3x)\cdot 3+...+f'(2015x)\cdot 2015]}{1}$$

$= f'(0) + 2f'(0) + 3f'(0) + ... + 2015f'(0)$

$= (1 + 2 + 3 + ... + 2015)\, f'(0)$

$=\frac{2015\cdot(2016)}{2}\cdot 2 \quad [\because f'(0) = 2]$

$= 2015 \times 2016$

67. (a) : $\because \underline{|17} = (1)\cdot(2)\cdot(3)...(9)(10)(11)(12)....(17)$

$\therefore$ It must be divisible by 9 and 11.

For divisibility by 9, sum of all digits must be a multiple of 9.

$\therefore$ $3 + 5 + 5 + 6 + x + y + 4 + 2 + 8 + 9 + 6 = 48 + x + y$ must be a multiple of 9 for which

$x + y = 6$ or 15 ... (1)

For divisibility by 11, we know that difference of sum of odd places digits and sum of even places digits must be either zero or a multiple of 11.

Let $a = 3 + 5 + x + 4 + 8 + 9 = 29 + x$

and $b = 5 + 6 + y + 2 + 0 + 6 = 19 + y$

$\because$ $a - b = 10 + x - y$

$\therefore$ $x - y = 1$ is possible only ... (2)

But, $x + y = 6$ and $x - y = 1$ can't give integral value.

Hence, $x + y = 15$ only.

68. (d) : $f(1) + 2f(2) + 3f(3) + ... + nf(n)$

$= n(n + 1)\, f(n)$ ... (i) for $n \geq 2$

Putting $n = n + 1$, we get

$f(1) + 2f(2) + 3f(3) + ... + nf(n) + (n + 1)\, f(n + 1)$

$= (n + 1)(n + 2)\, f(n + 1)$ ... (ii)

(ii) – (i) : $(n + 1)\, f(n + 1) = (n + 1)(n + 2)\, f(n + 1) - n(n + 1)\, f(n)$

$\Rightarrow$ $f(n + 1) = (n + 2)\, f(n + 1) - n\, f(n)$

$\Rightarrow$ $(n + 1)\, f(n + 1) = n\, f(n)$

$\Rightarrow$ $2f(2) = 3f(3) = 4\, f(4) ..... = nf(n)$

From (i),

$$f(1)+\underbrace{nf(n)+nf(n)+nf(n)+....+nf(n)}_{(n-1)\text{ times}}=n(n+1)f(n)$$

$\Rightarrow$ $1 + (n - 1)\, n\, f(n) = n(n + 1)\, f(n)$

$\Rightarrow$ $2n\, f(n) = 1$

$\therefore$ $f(n)=\frac{1}{2n}$ $\Rightarrow$ $f(500)=\frac{1}{1000}$

69. (a) : Without any restriction : 5^5

Exactly one empty cell can be selected in ${}^5C_1$ ways.

$\therefore$ Required probability

$$=\frac{{}^5C_1(4^5-{}^4C_1\cdot 3^5+{}^4C_2\cdot 2^5-{}^4C_3\cdot 1^5)}{5^5}=\frac{48}{125}$$

70. (c) : S = smoker, S' = Non-smoker, D = death by cancer

Using conditional probability, we can write

$P(D) = P(S)\, P(D/S)$ or $P(S')\, P(D/S')$

$$0.006=\frac{20}{100}\cdot P(D/S)+\frac{80}{100}\cdot P(D/S')$$

$$=\frac{1}{5}\cdot x+\frac{4}{5}\cdot\frac{x}{10}$$

[Let $P(D/S) = x$ and given $P(D/S) = 10\cdot P(D/S')$]

$\Rightarrow$ $x=\frac{3}{140}$

71. (a, b, d) : $\because$ $\cos x$ and $\sin x$ are solutions of

$$a_0\frac{d^2y}{dx^2}+a_1\frac{dy}{dx}+a_2y=0,$$

which is a differential equation of order 2 and degree 1.

$\therefore$ $y = A\cos x + B\sin x$ will be a general solution.

Further,

$$A\cos\left(x+\frac{\pi}{4}\right)\text{ and }A\cos\left(x+\frac{\pi}{4}\right)+B\sin\left(x-\frac{\pi}{4}\right)$$

both can also be expressed in $A\cos x + B\sin x$ form.

72. (a, c) : If $0 < n < 1$ ... (i) and $0<\theta<\frac{\pi}{2}$ ... (ii)

then $0<n\theta<\frac{\pi}{2}$

$\therefore$ If n is a fraction, $n\theta$ will be away from $\pi/2$ and so $\cos n\theta$ will be away from $\cos\frac{\pi}{2}$ *i.e.* 0

whereas $\cos^n\theta = (\cos\theta)^n$ may be very close to 0.

$\therefore$ In $0<\theta<\frac{\pi}{2}$, $\cos^n\theta\leq\cos n\theta$, if $0 < n < 1$

Hence, $(\cos\theta)^{1/2}\leq\cos\frac{1}{2}\theta$ and $(\cos\theta)^{5/6}\leq\cos\frac{5\theta}{6}$

73. (c) : Only option (c) is always false.

74. (b, d) : $\because$ $xy = 1$

$\therefore$ $y=\frac{1}{x}$ $\Rightarrow$ $\frac{dy}{dx}=-\frac{1}{x^2}$

$\therefore$ Slope of normal $= x_1^2 > 0$ at any point $P(x_1, y_1)$

If $(a-1)x - by + 4 = 0$ is a normal

then its slope $= \dfrac{a-1}{b} > 0$

$\therefore$ $a - 1$ and b must be of opposite sign.

$\Rightarrow$ $a > 1, b < 0$ or $a < 1, b > 0$

75. (b, c) : $\because$ $f(x)$ is continuous and differentiable on $[a, b]$ and $f(a) = 0 = f(b)$

$\therefore$ From Rolle's theorem,

$f'(c) = 0$ for some $c \in (a, b)$

i.e. $f'(x) = 0$ for some $x \in (a, b)$

Further, $f'(a) = f'(c)$ [$\because$ each $= 0$]

$f'(x)$ is continuous and differentiable in $[a, b]$

$\therefore$ From Rolle's theorem again,

$f''(x) = 0$ for some $x \in (a, c)$ *i.e.* $x \in (a, b)$

76. (b, c) : $\because$ $0 \in R$ but, $0 \cdot 0 > 0$ is not true

i.e. $(0, 0) \notin \rho$

$\therefore$ ρ is not reflexive

$\because$ $xy > 0 \Rightarrow yx > 0 \Rightarrow \rho$ is symmetric

Let $xy > 0$ and $yz > 0$

$\therefore$ $(xy)(yz) > 0 \Rightarrow (xz)y^2 > 0 \Rightarrow xz > 0$ [$\because y^2 > 0$]

$\therefore$ $x \rho y$ and $y \rho z \Rightarrow x \rho z$

$\therefore$ ρ is transitive.

77. (c, d): P (at least one) $= 1 - P$ (none)

Let n be number of parts produced

$\therefore$ $1-(0.95)^n \geq \dfrac{1}{2}$ will imply $(0.95)^n \leq \dfrac{1}{2}$

$\Rightarrow$ $n \log_{10} 0.95 \leq \log_{10} \dfrac{1}{2}$

$\Rightarrow$ $n(1.977 - 2) \leq -0.3$

$\Rightarrow$ $n \geq \dfrac{300}{23}$ $\therefore$ $n = 14$ or 15

78. (a, d) : $\because$ $f(2x + 1) = f(x)$

Putting $n = \dfrac{n+1}{2}$, we get $f\left(2 \cdot \dfrac{x+1}{2} - 1\right) = f\left(\dfrac{x+1}{2}\right)$

$\Rightarrow$ $f(x) = f\left(\dfrac{x+1}{2}\right)$

$\therefore$ $f(x) = f\left(\dfrac{x+1}{2}\right) = f\left(\dfrac{\frac{x+1}{2}+1}{2}\right) = f\left(\dfrac{x+1+2}{2^2}\right)$

$= f\left(\dfrac{\frac{x+1+2}{2^2}+1}{2}\right) = f\left(\dfrac{x+1+2+2^2}{2^3}\right) = \ldots$

$= f\left\{\dfrac{x+1+2+2^2+\ldots+2^{n-1}}{2^n}\right\}$

$= f\left\{\dfrac{x}{2^n} + \dfrac{(2^n-1)}{2^n(2-1)}\right\} = f\left\{\dfrac{x}{2^n} + 1 - \dfrac{1}{2^n}\right\}$

$\therefore$ When $n \to \infty$, $\dfrac{x}{2^n} \to 0, \dfrac{1}{2^n} \to 0 \Rightarrow f(x) = f(1) = 1$

$\therefore$ $f(x)$ is constant for all $x \Rightarrow f(2) = 1$ also

79. (a, b, c) : We have $\dfrac{(x-1)^2}{3} - \dfrac{(y+2)^2}{16} = 1$

Here, $a = \sqrt{3}, b = 4$

$\therefore$ Length of transverse axis $= 2a = 2\sqrt{3}$

Length of latus rectum $= \dfrac{2b^2}{a} = \dfrac{2(16)}{\sqrt{3}} = \dfrac{32}{\sqrt{3}}$

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{19}{3}}$$

Equation of directrices are given by

$$x - 1 = \pm\frac{a}{e} \quad \textit{i.e.} \quad x = 1 \pm \frac{3}{\sqrt{19}}$$

80. (b, c) : $\because$ $f(x) = \left[\dfrac{1}{[x]}\right]$

$\therefore$ $f(x)$ is not defined when $[x] = 0$

i.e. when $0 \leq x < 1$

$\therefore$ $D_f = R - [0, 1) = (-\infty, 0) \cup [1, \infty)$

When $x = 1, f(x) = 1$

When $x > 1$, $0 < \dfrac{1}{[x]} < 1 \Rightarrow \left[\dfrac{1}{[x]}\right] = 0$

When $-1 \leq x < 0$, $[x] = -1$

$\Rightarrow \left[\dfrac{1}{[x]}\right] = -1$

When $-\infty < x < -1$, $[x] \leq -2$

$\Rightarrow$ $-1 < \dfrac{1}{[x]} < 0$

$\Rightarrow \left[\dfrac{1}{[x]}\right] = -1$

Hence, $R_f = \{0\} \cup \{-1\} \cup \{1\}$

Problems from AROUND THE WORLD

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1. Let $a_1, a_2, ..., a_{100}$ be an ordered set of numbers. At each move it is allowed to choose any two numbers a_n, a_m and change them to the numbers

$$\frac{a_n^2}{a_m} - \frac{n}{m}\left(\frac{a_m^2}{a_n} - a_m\right) \text{ and } \frac{a_m^2}{a_n} - \frac{m}{n}\left(\frac{a_n^2}{a_m} - a_n\right)$$

respectively. Determine if it is possible, starting with the set with $a_i = \frac{1}{5}$ for $i = 20, 40, 60, 80, 100$ and $a_i = 1$ otherwise, to obtain a set consisting of integers only.

2. Does there exist a bijection f of
(a) a plane with itself
(b) three-dimensional space with itself such that for any distinct points A, B, line AB and line $f(A)$ $f(B)$ are perpendicular?

3. Find all triplets (x, y, z) of natural numbers such that y is a prime number, y and 3 do not divide z, and $x^3 - y^3 = z^2$.

4. Prove that the equation $x^3 + y^3 + z^3 + t^3 = 1999$ has infinitely many integral solutions.

5. Determine the maximum value of λ such that if $f(x) = x^3 + ax^2 + bx + c$ is a cubic polynomial with all its roots non negative, then $f(x) \geq \lambda (x - a)^3$ for all $x \geq 0$. Find the equality condition.

6. Determine all functions $f: R - \{0, 1\} \to R$ satisfying the functional relation $f(x) + f\left(\frac{1}{1-x}\right) = \frac{2(1-2x)}{x(1-x)}$.

7. Show that $19^{93} - 13^{99}$ is a positive integer divisible by 162.

8. A sergeant appoints arbitrarily chosen 9 or 10 soldiers every night, out of a 33 member guard team, for duty. Determine the least number of days within which all guards can be on duty the same number of times. How can the guard duty be arranged to achieve this?

9. Find all primes p for which the quotient $\frac{(2^{p-1} - 1)}{p}$ is a square.

10. Prove that

$$1 < \frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + ... + \frac{1}{3001} < \frac{4}{3}$$

11. Three congruent circles have a common point O and lie inside a triangle such that each circle touches a pair of sides of the triangle. Prove that the incentre and the circumcentre of the triangle and the point O are collinear.

12. Find the number of permutations, $(P_1, P_2, ..., P_6)$, of $(1, 2, ..., 6)$ such that for any k, $1 \leq k \leq 5$, $(P_1, P_2, ..., P_k)$ does not form a permutation of $1, 2, ..., k$.
[That is, $P_1 \neq 1$; (P_1, P_2) is not a permutation of 1, 2, 3, etc.]

13. Let f be a bijective (one-one and onto) function from the set $A = \{1, 2, 3, ..., n\}$ to itself. Show that there is a positive integer $M > 1$ such that $f^M(i) = f(i)$ for each $i \in A$.
[f^M denotes the composite function $fofo$ ..... repeated M times.]

SOLUTIONS

1. Suppose we change

$$a_n \text{ to } a_n' = \frac{a_n^2}{a_m} - \frac{n}{m}\left(\frac{a_m^2}{a_n} - a_m\right) \text{ and}$$

$$a_m \text{ to } a_m' = \frac{a_m^2}{a_n} - \frac{m}{n}\left(\frac{a_n^2}{a_m} - a_n\right)$$

Observe that $\frac{a_n'}{n} + \frac{a_m'}{m}$ equals

$$\left[\left(\frac{1}{n}\cdot\frac{a_n^2}{a_m} - \frac{1}{m}\cdot\frac{a_m^2}{a_n}\right) + \frac{a_m}{m}\right] + \left[\left(\frac{1}{m}\cdot\frac{a_m^2}{a_n} - \frac{1}{n}\cdot\frac{a_n^2}{a_m}\right) + \frac{a_n}{n}\right]$$

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which equals $\frac{a_n}{n}+\frac{a_m}{m}$. Thus, the quantity $\sum_{i=1}^{100}\frac{a_i}{i}$ is invariant under the given operation. Originally, this sum equals $I_1=\sum_{i=1}^{99}\frac{a_i}{i}+\frac{1}{500}$.

When each of the numbers $\frac{a_1}{1},\frac{a_2}{2},...,\frac{a_{99}}{99}$ is written as a fraction in lowest terms, none of their denominators are divisible by 125. On the other hand, 125 does divide the denominator of $\frac{1}{500}$. Thus when written as a fraction in lowest terms, I_1 must have a denominator divisible by 125.

Now suppose, by way of contradiction, that we could make all the numbers equal to integers $b_1, b_2, ...,b_{100}$ in that order. Then in $I_2=\sum_{i=1}^{100}\frac{b_i}{i}$, the denominator of each of the fraction $\frac{b_i}{i}$ is not divisible by 125. Thus, when I_2 is written as a fraction in lowest terms, its denominator is not divisible by 125 either. Then I_2 cannot possibly equal I_1, a contradiction. Therefore, we can never obtain a set consisting of integers only.

2. (a) Yes: simply rotate the plane 90° about some axis perpendicular to it. For example, in the xy-plane, we could map each point (x, y) to the point $(y, -x)$.

(b) Suppose such a bijection existed. Label the three-dimensional space with x-, y-, and z-axes. Given any point $P = (x_0, y_0, z_0)$, we also view it as the vector p from $(0, 0, 0)$ to (x_0, y_0, z_0). The given condition says that $(a - b)\cdot(f(a) - f(b)) = 0$ for any vectors a, b.

Assume without loss of generality that f maps the origin to itself otherwise, $g(p) = f(p) - f(0)$ is still a bijection and still satisfies the above equation. Plugging $b = (0, 0, 0)$ into the equation above, we have $a\cdot f(a) = 0$ for all a. Then the above equation reduces to $a\cdot f(b) + b\cdot f(a) = 0$.

Given any vectors a,b,c and any reals m, n, we then have

$m(a\cdot f(b) + b\cdot f(a)) = 0$

$n(a\cdot f(c) + c\cdot f(a)) = 0$

$a\cdot f(mb + nc) + (mb + nc)\cdot f(a) = 0$.

Adding the first two equations and subtracting the third gives $a\cdot(m\,f(b) + n\,f(c) - f(mb + nc)) = 0$.

Because this must be true for any vector a, we must have $f(mb + nc) = m\,f(b) + n\,f(c)$.

Therefore f is linear, and it is determined by how it transforms the unit vectors $i = (1, 0, 0)$, $j = (0, 1, 0)$, and $k = (0, 0, 1)$. If $f(i) = (a_1, a_2, a_3)$, $f(j) = (b_1, b_2, b_3)$ and $f(k) = (c_1, c_2, c_3)$, then for a vector x, we have

$$f(x)=\begin{bmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{bmatrix}x$$

Applying $f(a)\cdot a = 0$ with $a = i, j, k$, we have

$a_1 = b_2 = c_3 = 0$

Then applying $a\cdot f(b) + b\cdot f(a)$ with $(a, b) = (i, j)$, (j, k), (k, i) we have $b_1 = -a_2$, $c_2 = -b_3$, $c_1 = -a_3$.

Setting $k_1 = c_2$, $k_2 = -c_1$, and $k_3 = b_1$, we find that

$f(k_1 i + k_2 j + k_3 k) = k_1 f(i) + k_2 f(j) + k_3 f(k) = 0$

Because f is injective and $f(0) = 0$, this implies that $k_1 = k_2 = k_3 = 0$.

Then $f(x) = 0$ for all x, contradicting the assumption that f was surjective. Therefore our original assumption was false, and no such bijection exists.

3. We rewrite the equation in the form

$(x - y)(x^2 + xy + y^2) = z^2$

Any common divisor of $x - y$ and $x^2 + xy + y^2$ also divides both z^2 and $(x^2 + xy + y^2) - (x + 2y)(x - y) = 3y^2$. Because z^2 and $3y^2$ are relatively prime by assumption, $x - y$ and $x^2 + xy + y^2$ must be relatively prime as well. Therefore, both $x - y$ and $x^2 + xy + y^2$ are perfect squares.

Writing $a=\sqrt{x-y}$, we have

$x^2 + xy + y^2 = (a^2 + y)^2 + (a^2 + y)y + y^2$

$= a^4 + 3a^2y + 3y^2$

and $4(x^2 + xy + y^2) = (2a^2 + 3y)^2 + 3y^2$

Writing $m=2\sqrt{x^2+xy+y^2}$ and $n=2a^2+3y$, we have

$m^2 = n^2 + 3y^2$ or $(m - n)(m + n) = 3y^2$,

so $(m - n, m + n) = (1, 3y^2)$, $(y, 3y)$ or $(3, y^2)$.

In the first case, $2n = 3y^2 - 1$ and $4a^2 = 2n - 6y = 3y^2 - 6y - 1$

Hence, $a^2 \equiv 2 \pmod 3$, which is impossible.

In the second case, $n = y < 2a^2 + 3y = n$, a contradiction.

In the third case, we have

$4a^2 = 2n - 6y = y^2 - 6y - 3 < (y - 3)^2$.

When $y \geq 10$ we have $y^2 - 6y - 3 > (y - 4)^2$.

Hence, we must actually have $y = 2$, 3, 5, or 7. In this case, we have $a=\frac{\sqrt{y^2-6y-3}}{2}$, which is real only when $y = 7$, $a = 1$, $x = y + a^2 = 8$ and $z = 13$. This yields the unique solution $(x, y, z) = (8, 7, 13)$.

4. Observe that $(m - n)^3 + (m + n)^3 = 2m^3 + 6mn^2$.

Now suppose we want a general solution of the form

$$(x, y, z, t)=\left(a-b, a+b, \frac{c}{2}-\frac{d}{2}, \frac{c}{2}+\frac{d}{2}\right)$$

for integers a, b and odd integers c, d. One simple solution to the given equation is

$(x, y, z, t) = (10, 10, -1, 0)$, so we try setting $a = 10$ and $c = -1$. Then

$$(x, y, z, t) = \left(10 - b, 10 + b, -\frac{1}{2} - \frac{d}{2}, -\frac{1}{2} + \frac{d}{2}\right)$$

is a solution exactly when

$$(2000 + 60b^2) - \frac{1 + 3d^2}{4} = 1999 \Leftrightarrow d^2 - 80b^2 = 1$$

The second equation is a Pell's equation with solution $(d_1, b_1) = (9, 1)$

We can generate infinitely many more solutions by setting $(d_{n+1}, b_{n+1}) = (9d_n + 80b_n, 9b_n + d_n)$ for $n = 1, 2, 3, ...$

This can be proven by induction, and it follows from a general recursion

$$(p_{n+1}, q_{n+1}) = (p_1p_n + q_1q_nD, p_1q_n + q_1p_n)$$

for generating solutions to $p^2 - Dq^2 = 1$ given a non trivial solution (p_1, q_1).

A quick check also shows that each d_n is odd. Thus because there are infinitely many solutions (b_n, d_n) to the Pell's equation (and with each d_n odd), there are infinitely many integral solutions

$$(x_n, y_n, z_n, t_n) = \left(10 - b_n, 10 + b_n, -\frac{1}{2} - \frac{d_n}{2}, -\frac{1}{2} + \frac{d_n}{2}\right)$$

to the original equation.

5. Let α, β, γ be the three roots. Without loss of generality, suppose that $0 \le \alpha \le \beta \le \gamma$. We have $x - a = x + \alpha + \beta + \gamma \ge 0$ and $f(x) = (x - \alpha)(x - \beta)(x - \gamma)$. If $0 \le x \le \alpha$, then (applying the AM-GM inequality to obtain the first inequality below)

$$-f(x) = (\alpha - x)(\beta - x)(\gamma - x) \le \frac{1}{27}(\alpha + \beta + \gamma - 3x)^3$$

$$\le \frac{1}{27}(x + \alpha + \beta + \gamma)^3 = \frac{1}{27}(x - a)^3$$

so that $f(x) \ge -\frac{1}{27}(x - a)^3$.

Equality holds exactly when $\alpha - x = \beta - x = \gamma - x$ in the first inequality and $\alpha + \beta + \gamma - 3x = x + \alpha + \beta + \gamma$ in the second; that is, when $x = 0$ and $\alpha = \beta = \gamma$.

If $\beta \le x \le \gamma$, then (again applying the AM-GM inequality to obtain the first inequality below)

$$-f(x) = (x - \alpha)(x - \beta)(\gamma - x) \le \frac{1}{27}(x + \gamma - \alpha - \beta)^3$$

$$\le \frac{1}{27}(x + \alpha + \beta + \gamma)^3 = \frac{1}{27}(x - a)^3$$

so that again $f(x) \ge -\frac{1}{27}(x - a)^3$.

Equality holds exactly when $x - \alpha = x - \beta = \gamma - x$ in the first inequality and $x + \gamma - \alpha - \beta = x + \alpha + \beta + \gamma$; that is, when $\alpha = \beta = 0$ and $\gamma = 2x$.

Finally, when $\alpha < x < \beta$ or $x > \gamma$ then

$$f(x) > 0 \ge -\frac{1}{27}(x - a)^3$$

Thus, $\lambda = -\frac{1}{27}$ works. From the above reasoning we can find that λ must be at most $-\frac{1}{27}$ or else the inequality fails for the polynomial $f(x) = x^2(x - 1)$ at $x = \frac{1}{2}$.

Equality occurs either when $\alpha = \beta = \gamma$ and $x = 0$; or when $\alpha = \beta = 0$, γ is any non negative real, and $x = \frac{\gamma}{2}$.

6. The given function relation can be written as

$$f(x) + f\left(\frac{1}{1 - x}\right) = \frac{2}{x} + \frac{2}{x - 1} \quad ...(1)$$

Let t be any real number other than 0 and 1. Put $x = \frac{1}{1 - t}$ in (1); then $\frac{1}{1 - x} = 1 - \frac{1}{t}$ and we have

$$f\left(\frac{1}{1 - t}\right) + f\left(1 - \frac{1}{t}\right) = -2t + \frac{2}{t} \quad ...(2)$$

When $x = 1 - \frac{1}{t}$, $\frac{1}{1 - x} = t$ and now (1) yields

$$f\left(1 - \frac{1}{t}\right) + f(t) = \frac{2t}{t - 1} - 2t \quad ...(3)$$

Also replacing x by t in (1), we get

$$f(t) + f\left(\frac{1}{1 - t}\right) = \frac{2}{t} + \frac{2}{t - 1} \quad ..(4)$$

Now adding (3) and (4) and subtracting (2), we obtain

$$f(t) = \frac{t + 1}{t - 1}$$

7. In the binomial expansion of $\left(1 + \frac{6}{13}\right)^{15}$, the third term $\frac{15 \cdot 14}{1 \cdot 2}\left(\frac{6}{13}\right)^2 > 13$.

Therefore $\left(1 + \frac{6}{13}\right)^{15} > 13 \Rightarrow \left(1 + \frac{6}{13}\right)^{31} > 13^2$

$\Rightarrow 19^{31} > 13^{33} \Rightarrow 19^{93} > 13^{99}$

In the binomial expansion of $(1 + 18)^{93}$ the first term is 1; the second term is 93. $18 \equiv 12 \cdot 18 \pmod{81}$; all other terms are divisible by 81. In the expansion of $(1 + 12)^{99}$ the first term is 1; the second term is 99. $12 \equiv 18.\ 12 \pmod{81}$; the next two terms $\frac{99 \cdot 98}{1 \cdot 2} \cdot 12^2, \frac{99 \cdot 98 \cdot 97}{1 \cdot 2 \cdot 3} 12^3$ are divisible by 81; the other terms being multiples of 12^4 are also divisible by 81. Hence we have $19^{93} \equiv 13^{99} \pmod{81}$.

$\therefore$ $162 \mid 19^{93} - 13^{99}$ ($\because$ $19^{93} - 13^{99}$ is even)

8. Let x (resp. y) be the number of days on which 9 (resp. 10) soldiers do guard duty. Let k be the number of days on which every soldier gets duty. Then

$9x + 10y = 33k$ ...(1)

So we have to minimize $x + y$ subject to (1). Obviously when $k = 1$, (1) has no non - negative integral solutions for x, y. So $k \geq 2$, then $10(x + y) \geq 9x + 10y = 33k \geq 66 \Rightarrow x + y \geq 7x + y$ attains this minimum when $k = 2$; in that case, $y = 3$, and $x = 4$.

Here is one way of allocating the duty to the soldiers.

Let $D_1 = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$D_2 = \{10, 11, 12, 13, 14, 15, 16, 17, 18\}$,

$D_3 = \{19, 20, 21, 22, 23, 24, 25, 26, 27\}$,

$D_4 = \{28, 29, 30, 31, 32, 33, 1, 2, 3\}$,

$D_5 = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$,

$D_6 = \{14, 15, 16, 17, 18, 19, 20, 21, 22, 23\}$

and $D_7 = \{24, 25, 26, 27, 28, 29, 30, 31, 32, 33\}$

For $k = 1,..., 7$, and $j = 1,...,33$, the soldier S_j assigned duty on k^{th} day if $j \in D_k$.

9. For $p = 2$, the given quotient is not an even integer and so we can assume p is an odd prime. Then by Fermat's Little theorem $2^{p-1} - 1$ is divisible by p. Suppose that for some integer a, $2^{p-1} - 1 = pa^2$. Since p is odd we have that

$$\left(2^{\frac{(p-1)}{2}} - 1\right)\left(2^{\frac{(p-1)}{2}} + 1\right) = pa^2$$

Both the factors on the L.H.S. are odd and hence they are relatively prime to each other which implies that p divides exactly one of the two factors. Therefore either

$$2^{\frac{(p-1)}{2}} - 1 = px^2,\ 2^{\frac{(p-1)}{2}} + 1 = y^2$$

or $$2^{\frac{(p-1)}{2}} - 1 = x^2,\ 2^{\frac{(p-1)}{2}} + 1 = py^2$$

Case – I: Suppose $2^{\frac{(p-1)}{2}} - 1 = px^2,\ 2^{\frac{(p-1)}{2}} + 1 = y^2$.

But then $2^{\frac{(p-1)}{2}} = (y-1)(y+1)$ for which the only solution is $y = 3$ *i.e.*, $p = 7$. When $p = 7$ observe that the given quotient is indeed a square.

Case – II: Suppose $2^{\frac{(p-1)}{2}} - 1 = x^2,\ 2^{\frac{(p-1)}{2}} + 1 = py^2$.

Then if $p > 3$ we get that x^2 leaves remainder 3 when divided by 4 which is impossible. Thus p can only be equal to 3; when $p = 3$ also we observe that the given quotient is indeed a square.

Thus the primes p for which the quotient $\dfrac{(2^{p-1} - 1)}{p}$ is a square are $p = 3, 7$.

10. Consider 2001 numbers $\dfrac{1}{k}$, $1001 \leq k \leq 3001$

Using AM-HM inequality, we get

$$\left(\sum_{k=1001}^{3001} k\right)\left(\sum_{k=1001}^{3001} \frac{1}{k}\right) > (2001)^2$$

But $\displaystyle\sum_{k=1001}^{3001} k = (2001)^2$

Hence we get the inequality

$$\sum_{k=1001}^{3001} \frac{1}{k} > 1$$

On the other hand, grouping 500 terms at a time, we also have

$$S = \sum_{k=1001}^{3001} \frac{1}{k}$$

$$< \frac{500}{1000} + \frac{500}{1500} + \frac{500}{2000} + \frac{500}{2500} + \frac{1}{3001}$$

$$< \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{3000}$$

$$= \frac{3851}{3000} < \frac{4}{3}$$

Note: We can sharpen the above inequality. Consider the sum

$$S = \sum_{k=n+1}^{3n+1} \frac{1}{k}$$

There are $2n + 1$ terms in the sum and the middle term is $\dfrac{1}{2n+1}$. We can write the sum in the form

$$S = \frac{1}{2n+1} + \sum_{k=1}^{n}\left(\frac{1}{2n+1+k} + \frac{1}{2n+1-k}\right)$$

$$= \frac{1}{2n+1} + \frac{2}{(2n+1)} + \sum_{k=1}^{n} \frac{1}{1-\left(\dfrac{k}{2n+1}\right)^2}$$

For $0 < a < \dfrac{1}{2}$, we have

$$1 + a < \frac{1}{1-a} < 1 + 2a$$

Thus we get the bounds

$$\frac{1}{2n+1}+\frac{2}{2n+1}\sum_{k=1}^{n}\left[1+\left(\frac{k}{2n+1}\right)^2\right]<S$$

and $S<\frac{1}{2n+1}+\frac{2}{2n+1}\sum_{k=1}^{n}\left[1+2\left(\frac{k}{2n+1}\right)^2\right]$

This on simplification gives

$$1+\frac{2}{(2n+1)^3}\sum_{k=1}^{n}k^2<S<1+\frac{4}{(2n+1)^3}\sum_{k=1}^{n}k^2$$

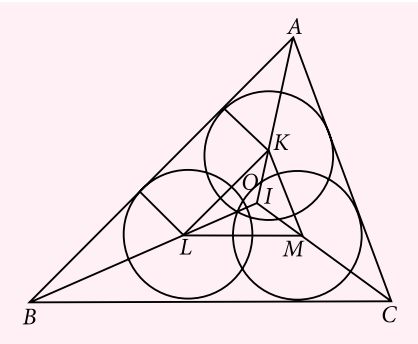
Now using the identity $\sum_{k=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$, the inequality simplifies to

$$1+\frac{n(n+1)}{3(2n+1)^2}<S<1+\frac{2}{3}\frac{n(n+1)}{(2n+1)^2}$$

But for $n\geq 1$, we also have $\frac{2}{9}\leq\frac{n(n+1)}{(2n+1)^2}\leq\frac{1}{4}$

This leads to $\frac{29}{27}<S<\frac{7}{6}$

11. Let K, L, M be the centres of the three circles of equal radii, meeting in a common point O, and pairwise touching some side of the triangle ABC in which they lie. Since the three circles have equal radii, we see that O is equidistant from the points K, L, M and so is the circumcentre of the triangle KLM. Also for same reason, the sides of the triangle KLM are parallel to the corresponding sides of the triangle ABC. (For instance, KL is parallel to AB, as K and L are equidistant from AB.)



Further AK, BL, CM are the bisectors of angles A, B, C of triangle ABC. So not only they meet in I, the incentre of the triangle ABC but also KI, LI, MI are the bisectors of the angles of triangle KLM, implying that I is also the incentre of triangle KLM.

It follows, from the relation $\frac{IK}{IA}=\frac{IL}{IB}=\frac{IM}{IC}$ that triangle KLM is homothetic to triangle ABC with respect to I, the centre of homothety. (This simply means that triangle ABC is a dilation (or an enlargement) of triangle KLM as seen from I).

A property of homothety is that the centre of homothety and any two corresponding points of the homothetic figures are collinear. Here, in particular, I and the circumcentres of triangles KLM and ABC have to be collinear. This is precisely what was to be proved.

12. We shall look at the problem from a general viewpoint. For any positive integer n, let T_n denote the number of permutations of $(P_1, P_2, P_3, \ldots, P_n)$ of 1, 2, 3, ..., n such that for each k, $1\leq k\leq n$, $(P_1, P_2, P_3, \ldots P_n)$ is not a permutation of 1, 2, ..., k. We shall obtain a formula for T_n which expresses T_n in terms of T_1, T_2, ... , T_{n-1} ($n>1$). (Such a relation is called a recurrence relation for T_n.)

Consider any permutation $(P_1, P_2, P_3, \ldots, P_n)$ of 1, 2, ..., n. There is always a least positive integer k such that $(P_1, P_2, P_3, \ldots P_k)$ is a permutation of 1, 2, ..., k. In fact k may be any integer in the set {1, 2, ..., n}; and those permutations for which $k=n$ are (1, 2, ..., n) for all of which k is the least positive integer satisfying the above property is T_k. $(n-k)!$, by our definition of T_n. The second factor corresponds to the permutations of $k+1$, $k+2$, ..., n which fill up the remaining $(n-k)$ places. Since there are $n!$ permutations in all, we obtain

$$n!=\sum_{k=1}^{n}T_k.(n-k)!$$

$$=T_1.(n-1)!+T_2.(n-2)!+\ldots\ldots+T_{n-1}.1!+T_n.0!$$

Hence

$T_n=n!-T_1.(n-1)!-T_2.(n-2)!\ldots.-T_{n-1}.1!$

Clearly $T_1=1$

$T_2=2!-T_1.1!=2-1=1$

$T_3=3!-T_1.1!-T_2.1!=6-2-1=3$

$T_4=4!-T_1.1!-T_2.2!-T_3.1!=24-6-2-3=13.$

$T_4=5!-T_1.4!-T_2.3!-T_3.2!-T_4.1!$

$=120-24-6-6-13=71$

$T_6=6!-T_1.5!-T_2.4!-T_3.3!-T_4.2!-T_5.1!$

$=720-120-24-18-26-71=461.$

Thus the required number is 461.

13. **1st Solution:**

i) If $f: A\to A$ is a bijective function then there is a unique bijective function $g: A\to A$ such that

$fog = gof = I_A$, then identity function on A. The function g is called the inverse of f and is denoted by f^{-1}. Thus, $fof^{-1} = I_A = f^{-1}of$

ii) $fo\ I_A = f = I_A\ of$

iii) If f and g are bijections from A to A, then so are gof and fog.

iv) If f, g, h are bijective functions from A to A and $fog = foh$, then $g = h$.

Apply f^{-1} at lef to both sides to obtain $g = h$.

Coming to the problem, since A has n elements, we see that there are only finitely many (in fact, $n!$) bijective functions from A to A as each bijective function f gives a permutation of $\{1, 2, 3, ..., n\}$ by taking $\{f(1), f(2), ..., f(n)\}$. Since f is a bijective function from A to A, so is each of the function in the sequence:

$$fof = f^2, fofof = f^3, ...f^n, ...$$

All these cannot be distinct, Since there are only finitely many bijective functions from A to A. Hence for some two distinct positive integers m and n, $m > n$ say, we must have $f^m = f^n$

If $n = 1$, we take $M = m$, to obtain the result. If $n > 1$, multiply both sides by $(f^{-1})^{n-1}$, to get $f^{m-n+1} = f$. We take $M = m - n + 1$ to get the relation

$$f^M = f, (M > 1)$$

Note : this means

$$f^M(i) = f(i) \text{ for all } i \in A.$$

2nd Solution: Take any element r in the set A and consider the sequence of elements

$$r, f(r), (fof)(r), (fofof)(r), ...$$

obtained by applying f successively. Since A has only n elements there must be repetitions in the above sequence. But when the first repetition occurs, this must be r itself; for, if the above sequence looks (for instance) like

$$r, a, b, c, d, e, c, ...$$

where the first repetition is an element c other than r, this would imply

$$f(b) = c \text{ and } f(e) = c,$$

contradicting the fact that f is a bijection. Thus for some positive integer $l_r \geq 1$, we have $f^{l_r}(r) = r$

This is true for each r in the set $A = 1, 2, ..., n$. By taking M to be the l.c.m. of $l_1, l_2, ..., l_r$ we get

$$f^M(r) = r \text{ for each } r \in A.$$

[Note: If f itself is the identity function the above proof fails because each $l_r = 1$. But in this case we may take M to be any integer greater than or equal to 2].

❍❍

1. who can participate

If you have taken any of the exams given below and possess plenty of grey cells, photographic memory then you are the right candidate for this contest. All you have to do is write down as many questions (with all choices) you can remember, neatly on a paper with name of the exam, your name, address, age, your photograph and mail them to us.

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WBJEE MOCK TEST PAPER

UNIT-I : ALGEBRA

The entire syllabus of Mathematics of WB-JEE is being divided in to six units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issue.

CATEGORY – I

For each correct one mark will be awarded, whereas, for each wrong answer, 25% of total marks (1/4) will be deducted. If candidates mark more than one answer, negative marking will be done.

1. Let x_1 and y_1 be real numbers. If z_1 and z_2 are complex numbers such that $|z_1| = |z_2| = 4$, then $|x_1z_1 - y_1z_2|^2 + |y_1z_1 + x_1z_2|^2 =$

(a) $32(x_1^2 + y_1^2)$ (b) $16(x_1^2 + y_1^2)$

(c) $4(x_1^2 + y_1^2)$ (d) 32

2. Let z_1 and z_2 be complex numbers such that $z_1 + i(\bar{z}_2) = 0$ and $\arg(\bar{z}_1 z_2) = \frac{\pi}{3}$. Then $\arg(\bar{z}_1)$ is

(a) $\frac{\pi}{3}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{5\pi}{12}$

3. Let the complex numbers z_1, z_2, z_3 and z_4 denote the vertices of a square taken in order. If $z_1 = 3 + 4i$ and $z_3 = 5 + 6i$, then the other two vertices z_2 and z_4 are respectively

(a) $5 + 4i$, $5 + 6i$ (b) $5 + 4i$, $3 + 6i$

(c) $5 + 6i$, $3 + 5i$ (d) $3 + 6i$, $5 + 3i$

4. Let α and β are the roots of the equation $px^2 + qx + r = 0$. If p, q, r are in A.P. and $\alpha + \beta = 4$, then $\alpha\beta =$

(a) -9 (b) 9 (c) 5 (d) -5

5. If α is a root of the equation $x^2 - 3x + 1 = 0$ then the value of $\frac{\alpha^3}{\alpha^6 + 1} =$

(a) $\frac{1}{14}$ (b) $\frac{1}{15}$ (c) $\frac{1}{16}$ (d) $\frac{1}{18}$

6. The quadratic equation $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ has equal roots, if

(a) $a \neq b, b = c$ (b) $a = b, b \neq c$

(c) $a \neq b, b \neq c$ (d) $a = b = c$

7. If $a_1 = 4$ and $a_{n+1} = a_n + 4n$ for $n \geq 1$, then the value of a_{100} is

(a) 19804 (b) 18904

(c) 18894 (d) 19904

8. If the first term of a G.P. is 729 and its 7^{th} term is 64, then the sum of first seven terms is

(a) 2187 (b) 2059 (c) 1458 (d) 2123

9. Let a_1, a_2, a_3, a_4 be in A.P. If $a_1 + a_4 = 10$ and $a_2a_3 = 24$, then the least term of them is

(a) 1 (b) 2 (c) 3 (d) 4

10. An A.P. has the property that the sum of first ten terms is half the sum of the next ten terms. If the second term is 13, then the common difference is

(a) 3 (b) 2 (c) 5 (d) 4

11. If ${}^nC_{r-1} = 36$ and ${}^nC_r = 84$ then

(a) $13r - 3n - 3 = 0$ (b) $10r - 3n - 30 = 0$

(c) $10r + 3n - 3 = 0$ (d) $10r - 3n - 3 = 0$

12. If $8!\left[\frac{1}{3!} + \frac{5}{4!}\right] = {}^9P_r$, then the value of r is

(a) 4 (b) 5 (c) 3 (d) 2

13. The number of onto functions from the set $\{1, 2, ..., 11\}$ to the set $\{1, 2, ..., 10\}$ is

(a) $5 \times 11!$ (b) $10!$

(c) $\frac{11!}{2}$ (d) $10 \times 11!$

By : Sankar Ghosh, HOD(Math), Takshyashila. Mob : 09831244397.

14. Let n be a positive even integer. The ratio of the largest coefficient and the 2nd largest coefficient in the expression of $(1 + x)^n$ is 11 : 10. Then the number of terms in the expansion of $(1 + x)^n$ is

(a) 20 (b) 21 (c) 10 (d) 11

15. If $(1 + x + x^2)^n = 1 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, then $2a_1 - 3a_2 + ... -(2n + 1)a_{2n} =$

(a) n (b) $-n$ (c) $n + 1$ (d) $-n + 1$

16. The value of n if $C_0 + 2C_1 + 3C_2 + ... + (n + 1)C_n = 576$, is

(a) 7 (b) 5 (c) 6 (d) 9

17. Let A and B are square matrices of order 'n' such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be true?

(a) Either A or B is zero matrix
(b) $A = B$
(c) $AB = BA$
(d) Either A or B is an identity matrix.

18. If the matrix $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} = A + B$, where A is symmetric and B is skew symmetric, then $B =$

(a) $\begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

19. The value of the determinant

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$ is equal to

(a) 0 (b) $1 + a^2 + b^2$
(c) $(1 + a + b^2)^3$ (d) $(1 + a^2 + b^2)^3$

20. If the determinant of the adjoint of a (real) matrix of order 3 is 25, then the determinant of the inverse of the matrix is

(a) 0.2 (b) ± 5 (c) $\frac{1}{\sqrt[5]{625}}$ (d) ± 0.2

21. If $n(A) = 43$, $n(B) = 51$ and $n(A \cup B) = 75$, then $n((A - B) \cup (B - A)) =$

(a) 53 (b) 45 (c) 56 (d) 66

22. If A and B are non-empty sets such that $A \supset B$, then

(a) $B' - A' = A - B$ (b) $B' - A' = B - A$
(c) $A' - B' = A - B$ (d) $A' \cap B' = B - A$

23. For any two real numbers A and B, we define aRb if and only if $\sin^2 a + \cos^2 b = 1$. The relation R is

(a) reflexive but not symmetric
(b) symmetric but not transitive
(c) transitive but not reflexive
(d) an equivalence relation

24. The identity mapping $I_c : S \to S$ is defined as $I_s(x) = x$ for $x \in S$. Suppose $f : A \to B$ is a bijection. Then which one of the following is true?

(a) $f^{-1}of \neq I_A$ but $fof^{-1} = I_B$
(b) $f^{-1}of = I_A$ and $fof^{-1} = I_B$
(c) $f^{-1}of = I_A$ but $fof^{-1} \neq I_B$
(d) $f^{-1}of \neq I_A$ and $fof^{-1} \neq I_B$

25. The value of x for which $\log_3 (5 \cdot 3^{x-1} + 1)$, $\log_9 (3^{1-x} + 1)$ and 1 are in A.P. is

(a) $\log_3\left(\frac{5}{3}\right)$ (b) $\log_3\left(\frac{3}{5}\right)$

(c) $\log_3\left(\frac{3}{2}\right)$ (d) $\log_3\left(\frac{2}{5}\right)$

26. $\log(\sin 1°) \times \log(\sin 2°) \times \log(\sin 3°)$ .....$\log(\sin 179°)$

(a) is positive
(b) is negative
(c) lies between 1 and 180
(d) zero

27. If $n = 2015!$ then the value of $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + ... + \frac{1}{\log_{2015} n}$ is

(a) $n!$ (b) $\log n!$
(c) 1 (d) none of these

28. If $\log_{0.3}(x + 1) \leq \log_{0.09}(x - 1)$, then

(a) $x < 2$ (b) $x > 2$
(c) $x \geq 2$ (d) none of these

29. If $\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$, then $P(n)$ is true for

(a) $n \geq 1$ (b) $n > 0$ (c) $n < 0$ (d) $n \geq 2$

30. If $P(n)$ is a statement $(n \in N)$ such that, if $P(K)$ is true, $P(K + 1)$ is true for $k \in N$, then $P(n)$ is true

(a) for all n (b) for all $n \geq 3$
(c) for all $n > 4$ (d) none of these

CATEGORY – II

For each correct answer 2 marks will be allotted, whereas, for each wrong answer, 25% of total marks (1/2) will be deducted.

31. If the sum of the n terms of an A.P. is $cn(n - 1)$ where $c \neq 0$, then the sum of the squares of these terms is

(a) $c^2n^2(n+1)^2$ (b) $\frac{2}{3}c^2n(n-1)(2n-1)$

(c) $\frac{2}{3}c^2n(n+1)(2n+1)$ (d) $\frac{1}{3}c^2n^2$

32. Let c be a complex number such that $|c| = 1$. If the equation $cz^2 + z + 1 = 0$ has purely imaginary root, then $\tan(\arg c) =$

(a) $\frac{\sqrt{5}-1}{2}$ (b) $\frac{\sqrt{5}+1}{2}$

(c) $\sqrt{\frac{\sqrt{5}-1}{2}}$ (d) $\sqrt{\frac{\sqrt{5}+1}{2}}$

33. If $A=\begin{bmatrix}3 & 7\\ 1 & 2\end{bmatrix}$ then the value of the determinant $|A^{2015} - 3A^{2014}| =$

(a) 8 (b) −8 (c) 7 (d) −7

34. If $A=\begin{bmatrix}\sin\alpha & -\cos\alpha & 0\\ \cos\alpha & \sin\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$ then A^{-1} is equal to

(a) A (b) I

(c) A^T (d) none of these

35. A five digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways in which this can be done is

(a) 216 (b) 240 (c) 600 (d) 3125

CATEGORY–III

In this section more than 1 answer can be correct. Candidates will have to mark all the correct answers, for which 2 marks will be awarded. If candidates marks one correct and one incorrect answer then no marks will be awarded. But if, candidates marks only correct, without making any incorrect, formula below will be used to allot marks.
2 × (no. of correct response/total no. of correct options).

36. Let $A = \{1, 2, 3\}$, we define $R = \{(1, 1), (2, 2), (3, 3)\}$ then it is

(a) reflexive
(b) symmetric
(c) ordered relation on A.
(d) none of these

37. If $f(x) = 2x - \sin x$ and $g(x) = \sqrt[3]{x}$, then

(a) range of gof is R
(b) gof is one-one
(c) both f and g are one-one
(d) both f and g are onto.

38. If $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}$, then

(a) $A^2 = I$ (b) $B^2 = I$

(c) $A^2 = -I$ (d) $B^2 = -I$

39. Let $p\lambda^4+q\lambda^3+r\lambda^2+s\lambda+t=\begin{vmatrix}\lambda^2+3\lambda & \lambda-1 & \lambda-3\\ \lambda-1 & -2\lambda & \lambda-4\\ \lambda-3 & \lambda+3 & 3\lambda\end{vmatrix}$

be an identity in λ, where p, q, r, s and t are constants. Then the value of t is

(a) −3 (b) 3 (c) −6 (d) 6

40. If $10! = 2^p \cdot 3^q \cdot 5^r \cdot 7^s$, then

(a) $p = 2q$ (b) $pqrs = 64$

(c) number of divisors of 10! is 280

(d) number of ways of putting 10! as a product of two number is 135

SOLUTIONS

1. (a) : $|x_1z_1 - y_1z_2|^2 + |y_1z_1 + x_1z_2|^2$

$= |x_1z_1|^2 + |y_1z_2|^2 - 2\text{Re}(x_1y_1z_1z_2) + |y_1z_1|^2 + |x_1z_2|^2 + 2\text{Re}(x_1y_1z_1z_2)$

$= |x_1z_1|^2 + |y_1z_2|^2 + |y_1z_1|^2 + |x_1z_2|^2$

$= x_1^2|z_1|^2 + y_1^2|z_2|^2 + y_1^2|z_1|^2 + x_1^2|z_2|^2$

$= x_1^2 4^2 + y_1^2 4^2 + y_1^2 4^2 + x_1^2 4^2 \quad [\because |z_1| = |z_2| = 4]$

$= 2(x_1^2 + y_1^2)4^2$

$= 32(x_1^2 + y_1^2)$

2. (d) : Given that $z_1 + i\bar{z}_2 = 0$

$\Rightarrow \overline{(z_1 + i\bar{z}_2)} = 0$

$\Rightarrow \bar{z}_1 - iz_2 = 0 \Rightarrow i\bar{z}_1 + z_2 = 0 \Rightarrow z_2 = -i\bar{z}_1$

Further, $\arg(\bar{z}_1 z_2) = \frac{\pi}{3}$

$\Rightarrow \arg(\bar{z}_1(-i\bar{z}_1)) = \frac{\pi}{3} \quad [\because z_2 = -i\bar{z}_1]$

$\Rightarrow \arg(\bar{z}_1) + \arg(-i) + \arg(\bar{z}_1) = \frac{\pi}{3}$

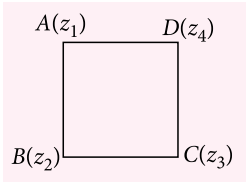
$\Rightarrow 2\arg(\bar{z}_1) - \frac{\pi}{2} = \frac{\pi}{3}$

$\Rightarrow \arg \bar{z}_1 = \frac{5\pi}{12}$

3. (b) : Co-ordinates of the points $A(z_1) = (3, 4)$ and $C(z_3) = (5, 6)$ respectively.

$\therefore$ Length of diagonal $= \sqrt{4+4} = \sqrt{8}$ units

The length of another diagonal must be $\sqrt{8}$
That is possible only when we take the other two vertices as $5 + 4i$ and $3 + 6i$.



4. (a) : Given that α, β are the roots of the equation $px^2 + qx + r = 0$

$\therefore \quad \alpha+\beta=-\frac{q}{p}$ and $\alpha\beta=\frac{r}{p}$

$\because \quad \alpha+\beta=4 \quad \therefore -\frac{q}{p}=4 \Rightarrow q=-4p$

Also, we have $2q = p+r \quad [\because p, q, r \in \text{A.P.}]$

$\therefore \quad 2(-4p) = p + r \Rightarrow r = -9p$

Thus $\alpha\beta=\frac{r}{p}=-\frac{9p}{p}=-9$

5. (d) : Given that α is a root of $x^2 - 3x + 1 = 0$

$\therefore \quad \alpha^2 - 3\alpha + 1 = 0 \Rightarrow \alpha^2 = 3\alpha - 1$

and $\alpha^6 + 1 = (\alpha^2 + 1)(\alpha^4 - \alpha^2 + 1)$

$= (3\alpha)(9\alpha^2 - 6\alpha + 1 - \alpha^2 + 1)$

$= 3\alpha(8\alpha^2 - 6\alpha + 2)$

$= 6\alpha(4\alpha^2 - 3\alpha + 1)$

Now $\frac{\alpha^3}{\alpha^6+1}=\frac{\alpha^3}{6\alpha(4\alpha^2-3\alpha+1)}=\frac{\alpha^2}{6(4\alpha^2-3\alpha+1)}$

$=\frac{\alpha^2}{6(4\alpha^2-\alpha^2)} \quad [\because \alpha^2=3\alpha-1]$

$=\frac{\alpha^2}{6\times3\alpha^2}=\frac{1}{18}$

6. (d) : The given equation is

$(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$

At $a = b = c = m$ (say), the above equation becomes

$(x - m)^2 = 0$

This is a quadratic equation of equal roots.

7. (a) : $a_1 = 4$

$a_2 = a_1 + 4 \times 1$

$a_3 = a_2 + 4 \times 2$

$a_4 = a_3 + 4 \times 3$

$\vdots$

$a_{100} = a_{99} + 4 \times 99$

Adding all the above equations, we get

$(a_1 + a_2 + a_3 + \ldots + a_{100}) = (4 + a_1 + a_2 + a_3 + \ldots + a_{99}) + 4(1 + 2 + 3 + \ldots + 99)$

$\Rightarrow \quad a_{100}=4+4\times\frac{99\times100}{2}=19804$

8. (b) : Let the 1st term = a and common ratio = r

Then $a_1 = 729$ and $a_7 = 64$

$\because \quad a_7 = a_1 r^6 \therefore 64 = 729\, r^6$

$\Rightarrow \quad r^6=\frac{64}{729}=\left(\frac{2}{3}\right)^6 \quad \therefore \quad r=\frac{2}{3}$

Now, $S_7=a_1\left(\frac{1-r^7}{1-r}\right)=729\left[\frac{1-\left(\frac{2}{3}\right)^7}{1-\frac{2}{3}}\right]=2059$

9. (b) : Let $a_1 = a - 3d$, $a_2 = a - d$, $a_3 = a + d$ and $a_4 = a + 3d$

Given that $a_1 + a_4 = 10 \Rightarrow 2a = 10 \quad \therefore \quad a = 5$

and $a_2 a_3 = 24 \Rightarrow a^2 - d^2 = 24 \quad \therefore \quad d^2 = 1$

$\Rightarrow \quad d = 1$

The A.P. is 2, 4, 6, 8.

10. (b) : Sum of first 10 terms

$=\frac{1}{2}$ (sum of next 10 terms)

$\Rightarrow \quad S_{10}=\frac{1}{2}(S_{20}-S_{10})$

$\Rightarrow \quad 3S_{10} = S_{20}$

$\Rightarrow \quad 3\left[\frac{10}{2}(2a+9d)\right]=\frac{20}{2}[2a+19d]$

$\Rightarrow \quad 2a = 11d$

$\Rightarrow \quad 2(13 - d) = 11d \quad [\because \text{second term is } 13 \therefore a = 13 - d]$

$\Rightarrow \quad 13d = 26 \Rightarrow d = 2$

11. (d) : ${}^nC_{r-1} = 36 \Rightarrow \frac{n!}{(n-r+1)!(r-1)!}=36$

$\Rightarrow \quad \frac{n!}{(n-r+1)(n-r)!(r-1)!}=36 \qquad \ldots(i)$

Again ${}^nC_r = 84 \Rightarrow \frac{n!}{r!(n-r)!}=84$

$\Rightarrow \quad \frac{n!}{r(r-1)!(n-r)!}=84 \qquad \ldots(ii)$

Dividing (i) by (ii), we get $\frac{r}{n-r+1}=\frac{36}{84}$

$\Rightarrow \quad 7r = 3n - 3r + 3 \Rightarrow 10r - 3n - 3 = 0$

12. (b) : $8!\left[\frac{1}{3!}+\frac{5}{4!}\right]={}^9P_r$

$\Rightarrow \quad 8!\left[\frac{4+5}{4!}\right]={}^9P_r$

$\Rightarrow \quad \frac{8!\times9}{4!}=\frac{9!}{(9-r)!}$

$\Rightarrow \quad (9 - r)! = 4! \Rightarrow r = 5$

13. (d) : Let $A = \{1, 2, 3, ..., 11\}$ and $B = \{1, 2, 3, ..., 10\}$

$\therefore \quad n(A) = 11, n(B) = 10$

For onto function, 9 elements in B has one pre-image in A and 1 element in B will have 2 pre-image in A.

(i) When each element in B has only one pre-image in A.

For 1st element in B we have 11 options in A

For second element in B we have 10 options in A

.....

For 10^{th} element in B we have 2 options in A

$\therefore$ Total number $= 11 \cdot 10 \cdot 9. \ldots\ldots \cdot 2 = \underline{|11}$

(ii) When one element in B has 2 pre-images in A.

Any one element in B can be chosen to have a second pre-image in A *i.e.* in 10 ways

$\therefore$ Required number of onto functions $= 10 \times \underline{|11}$

14. (b) : $\because$ n is a positive even integer

$\therefore \quad n = 2K$ (say), where $K \in N$

In $(1 + x)^n = (1 + x)^{2K}$, the largest and second largest co-efficients are ${}^{2K}C_K$ and ${}^{2K}C_{K-1}$ respectively.

$\therefore \quad {}^{2K}C_K : {}^{2K}C_{K-1} = 11 : 10$ (given)

$$\Rightarrow \quad \frac{\underline{|2K}}{\underline{|K}\,\underline{|K}} \cdot \frac{\underline{|K-1}\,\underline{|K+1}}{\underline{|2K}} = \frac{11}{10}$$

$$\Rightarrow \quad \frac{K+1}{K} = \frac{11}{10} \quad \Rightarrow \quad K = 10 \text{ and } n = 20$$

$\therefore$ Number of terms = 21

15. (b) : $(1 + x + x^2)^n = 1 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$ ...(i)

At $x = -1$, $1 = 1 - a_1 + a_2 - a_3 + ... + a_{2n}$ ....(ii)

Differentiating (i) with respect to x, we get

$n(1 + x + x^2)^{n-1}(1 + 2x) = a_1 + 2a_2x + 3a_3x^2 + ...+ 2na_{2n}x^{2n-1}$

At $x = -1$

$-n = a_1 - 2a_2 + 3a_3 - ..... - 2n\,a_{2n}$ ...(iii)

Subtracting (ii) from (iii), we get

$-n - 1 = -1 + 2a_1 - 3a_2 + 4a_3 - ..... -(2n + 1)a_{2n}$

$\Rightarrow \quad -n = 2a_1 - 3a_2 + 4a_3 - ..... - (2n + 1)a_{2n}$

16. (a) : $C_0 + 2C_1 + 3C_2 + ... + (n + 1)C_n$

$= (C_0 + C_1 + C_2 + ... + C_n) + (C_1 + 2C_2 + 3C_3 + ... + nC_n)$

$$= 2^n + \left(n + 2 \cdot \frac{n(n-1)}{2!} + 3 \cdot \frac{n(n-1)(n-2)}{3!} + ...+ n\right)$$

$$= 2^n + n\left(1 + n - 1 + \frac{(n-1)(n-2)}{2!} + ... + 1\right)$$

$= 2^n + n(1 + 1)^{n-1}$

$= 2^n + n \cdot 2^{n-1}$

$= (n + 2)2^{n-1}$

$\therefore \quad (n + 2)2^{n-1} = 576 = 2^6 \times 3^2 = (2 + 7)2^{7-1}$

$\therefore \quad n = 7$

17. (c) : $A^2 - B^2 = A^2 - BA + AB - B^2$

$\Rightarrow \quad 0 = -BA + AB \quad \Rightarrow \quad AB = BA$

Option (c) is the only necessary condition.

18. (d) : $B = \frac{1}{2}(A - A')$

$$= \frac{1}{2}\left[\begin{pmatrix} 2 & 3 \\ 5 & -1 \end{pmatrix} - \begin{pmatrix} 2 & 5 \\ 3 & -1 \end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

19. (d) :
$$\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix}$$

Applying $C'_1 \to C_1 - bC_3$ and $C'_2 = aC_3 + C_2$

$$= (1 + a^2 + b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1 - a^2 - b^2 \end{vmatrix}$$

$= (1 + a^2 + b^2)^2 [1(1 - a^2 - b^2 + 2a^2) - 2b(-b)]$

$= (1 + a^2 + b^2)^2 (1 + a^2 + b^2)$

$= (1 + a^2 + b^2)^3$

20. (d) : $|\text{adj } A| = 25, n = 3$

We have, $|\text{adj } A| = |A|^{n-1}$

$\therefore \quad 25 = |A|^2 \quad \Rightarrow \quad |A| = \pm 5$

$$\therefore \quad |A^{-1}| = \frac{1}{|A|} = \pm\frac{1}{5} = \pm 0.2$$

21. (c) : Let $n(A \cap B) = x$

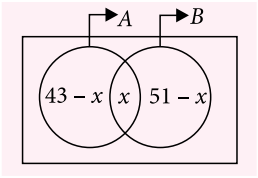
Now $n(A - B) + n(B - A) + n(A \cap B) = n(A \cup B)$

$\Rightarrow \quad 43 - x + 51 - x + x = 75$

$\Rightarrow \quad x = 19$

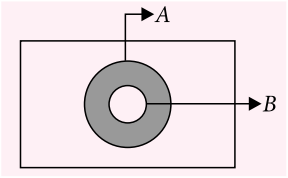
$\therefore \quad n((A - B) \cup (B - A)) = n(A - B) + n(B - A)$

$= 43 - x + 51 - x = 94 - 38 = 56$



22. (a) : $A \supset B$

$\therefore \quad B' - A' = A - B$



23. (d) : $\because \quad aRb$ if $\sin^2 a + \cos^2 b = 1$

$\because \quad \sin^2 a + \cos^2 a = 1 \ \forall \ a \in R$

$\therefore \quad aRa \Rightarrow R$ is reflexive.

$\because \sin^2 a + \cos^2 b = 1 \quad \Rightarrow \quad 1 - \cos^2 a + 1 - \sin^2 b = 1$

$\Rightarrow \quad \sin^2 b + \cos^2 a = 1 \quad \Rightarrow \quad bRa$

$\therefore \quad R$ is symmetric.

Further, let aRb and bRc

i.e. $\sin^2 a + \cos^2 b = 1$

and $\sin^2 b + \cos^2 c = 1$

$\Rightarrow \quad \sin^2 a + 1 + \cos^2 c = 2$

$\Rightarrow \quad \sin^2 a + \cos^2 c = 1 \quad \Rightarrow \quad aRc$

$\therefore \quad R$ is transitive.

$\therefore \quad R$ is an equivalence relation.

24. (b) : See inverse mapping

25. (b) : $\log_3(5\cdot 3^{x-1} + 1), \log_9(3^{1-x} + 1), 1$ are in A.P.

$\therefore \quad 2\log_9 (3^{1-x} + 1) = \log_3(5\cdot 3^{x-1} + 1) + 1$

$$\Rightarrow \quad \frac{2}{2}\log_3(3^{1-x} + 1) = \log_3(5\cdot 3^{x-1} + 1) + 1$$

$$\Rightarrow \quad \log_3\left(\frac{3^{1-x} + 1}{5\cdot 3^{x-1} + 1}\right) = \log_3 3$$

$$\Rightarrow \quad \frac{3^{1-x} + 1}{5\cdot 3^{x-1} + 1} = 3$$

$\Rightarrow \quad 5\cdot 3^x + 3 = 3\cdot 3^{-x} + 1$

$\Rightarrow \quad 5(3^x)^2 + 3\cdot 3^x = 3 + 3^x$

$\Rightarrow \quad 5(3^x)^2 + 2\cdot 3^x - 3 = 0$

$\Rightarrow \quad 5(3^x)^2 + 5\cdot 3^x - 3\cdot 3^x - 3 = 0$

$\Rightarrow \quad 5\cdot 3^x (3^x + 1) - 3(3^x + 1) = 0$

$\Rightarrow \quad (5\cdot 3^x - 3)(3^x + 1) = 0$

$$\Rightarrow \quad 5\cdot 3^x - 3 = 0 \quad \Rightarrow \quad 3^x = \frac{3}{5} \quad \therefore \quad x = \log_3\left(\frac{3}{5}\right)$$

Also $3^x + 1 = 0 \quad \therefore \quad 3^x = -1$ (not possible)

26. (d) : $\log(\sin 1°) \times \log(\sin 2°) \times \log(\sin 3°) \times \ldots\ldots \log(\sin 179°)$

$= \log(\sin 1°)\cdot\log(\sin 2°)\cdot\log(\sin 3°) \ldots\ldots\ldots \log(\sin 90°) \ldots\ldots \log(\sin 179°)$

$= \log(\sin 1°) \log(\sin 2°) \ldots\ldots \log 1 \ldots\ldots \log(\sin 179°) = 0$

27. (c) : $$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots\ldots + \frac{1}{\log_{2015} n}$$

$= \log_n 2 + \log_n 3 + \log_n 4 + \ldots\ldots + \log_n 2015$

$= \log_n (2\cdot 3\cdot 4\cdot \ldots\ldots\ldots 2015)$

$= \log_{2015!} (2015!) = 1$

28. (c) : $\log_{0.3}(x - 1) \le \log_{0.09}(x - 1)$

$$\Rightarrow \quad \frac{\log(x-1)}{\log 0.3} \le \frac{\log(x-1)}{\log 0.09} \Rightarrow \quad \frac{\log(x-1)}{\log 0.3} \le \frac{\log(x-1)}{2\log 0.3}$$

$\Rightarrow \quad \log_{0.3}(x - 1)^2 - \log_{0.3}(x - 1) \le 0$

$\Rightarrow \quad \log_{0.3}(x - 1) \le 0$

$$\Rightarrow \quad \frac{\log(x-1)}{\log 0.3} \le 0$$

$\Rightarrow \quad \log(x - 1) \ge 0 \quad [\because \log_{10} 0.3 = \log_{10} 3 - \log_{10} 10 < 0]$

$\Rightarrow \quad x - 1 \ge 10^0 \quad \Rightarrow \quad x - 1 \ge 1 \quad \Rightarrow \quad x \ge 2$

29. (d) : $P(n) : \dfrac{4^n}{n+1} < \dfrac{(2n)!}{(n!)^2}$

For $n = 1$, $P(n)$ is not true.

For $n = 2$, $P(2) : \dfrac{4^2}{2+1} < \dfrac{4!}{(2)^2} \Rightarrow \dfrac{16}{3} < \dfrac{24}{4}$ is true.

Let for $n = m > 2$, $P(n)$ is true *i.e* $\dfrac{4^m}{m+1} < \dfrac{(2m)!}{(m!)^2}$

Now $$\frac{4^{m+1}}{m+2} = \frac{4^m}{m+1}\cdot\frac{4(m+1)}{m+2} < \frac{(2m)!}{(m!)^2}\cdot\frac{4(m+1)}{m+2}$$

$$= \frac{(2m)!(2m+1)(2m+2)4(m+1)(m+1)^2}{(2m+1)(2m+2)(m!)^2(m+1)^2(m+2)}$$

$$= \frac{[2(m+1)]!}{[(m+1)!]^2}\cdot\frac{2(m+1)^2}{(2m+1)(m+2)}$$

$$< \frac{[2(m+1)]!}{[(m+1)!]^2} \qquad \left[\because \frac{2(m+1)^2}{(2m+1)(m+2)} < 1, \forall m > 0\right]$$

Hence, for $n \ge 2$, $P(n)$ is true.

30. (d) : Unless we prove $P(1)$ is true, nothing can be said.

31. (b) : Given that $\Sigma t_n = cn(n - 1) = S_n$ (say)

$\therefore \quad t_n = S_n - S_{n-1}$

$= cn(n - 1) - c(n - 1)(n - 2)$

$= c(n - 1)(n - n + 2)$

$t_n = 2c(n - 1)$

Now $t_n^2 = 4c^2(n - 1)^2$

$\therefore \quad \Sigma t_n^2 = 4c^2\Sigma(n^2 - 2n + 1)$

$= 4c^2[\Sigma n^2 - 2\Sigma n + n]$

$$= 4c^2\left[\frac{n(n+1)(2n+1)}{6} - \frac{2n(n+1)}{2} + n\right]$$

$$= 4c^2 n\left[\frac{2n^2 + 3n + 1 - 6n - 6 + 6}{6}\right]$$

$$= \frac{4c^2 n[2n^2 - 3n + 1]}{6}$$

$$= \frac{2}{3}c^2 n(n-1)(2n-1)$$

32. (d) : Let $z = iy$, $y \in R$ and $c = \cos\alpha + i\sin\alpha$

$\therefore \quad cz^2 + z + 1 = 0$

$\Rightarrow -(\cos\alpha + i\sin\alpha)y^2 + iy + 1 = 0$

$\Rightarrow (\cos\alpha + i\sin\alpha)y^2 - iy - 1 = 0$

$\Rightarrow (\cos\alpha \cdot y^2 - 1) + (\sin\alpha \cdot y^2 - y)i = 0$

$\Rightarrow y^2\cos\alpha - 1 = 0$ and $y^2\sin\alpha - y = 0$

$\Rightarrow y^2\cos\alpha = 1$ and $y\sin\alpha = 1$, $y \neq 0$

$\Rightarrow y^2\cos\alpha - y^2\sin^2\alpha = 0$

$\Rightarrow y^2(\sin^2\alpha - \cos\alpha) = 0$

$\therefore \quad 1 - \cos^2\alpha - \cos\alpha = 0 \Rightarrow \cos^2\alpha + \cos\alpha - 1 = 0$

$$\Rightarrow \cos\alpha = \frac{-1 \pm \sqrt{1^2 - 4\cdot 1(-1)}}{2\cdot 1} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\Rightarrow \cos\alpha = \frac{\sqrt{5}-1}{2} \quad \therefore \sec^2\alpha = \left(\frac{\sqrt{5}+1}{2}\right)^2$$

$$\therefore \tan^2\alpha = \left(\frac{\sqrt{5}+1}{2}\right)^2 - 1 = \frac{\sqrt{5}+1}{2}$$

$$\therefore \tan\alpha = \tan(\arg c) = \sqrt{\frac{\sqrt{5}+1}{2}}$$

33. (d) : $|A^{2015} - 3A^{2014}|$

$= |A^{2014}|\ |A - 3I|$

$= |A|^{2014}\ |A - 3I|$

$$= 1\begin{vmatrix} 0 & 7 \\ 1 & -1 \end{vmatrix} = 1(-7) = -7 \qquad \left[\because A = \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}\right]$$

34. (d) : Here $|A| = \begin{vmatrix} \sin\alpha & -\cos\alpha & 0 \\ \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$

By definition $A^{-1} = \frac{1}{|A|}\text{adj } A = \text{adj } A \quad (\because |A| = 1)$

$\therefore \quad A^{-1} = \text{adj } A$

35. (a) : Using 1, 2, 3, 4, 5, we can form 5 digits number divisible by 3 in $\lfloor 5$ ways. Now we fix '0' at unit place, tenth place, hundredth place and thousand place then number of ways = $4 \times 4!$

$\therefore \quad$ Total number of ways = 120 + 96 = 216

36. (a, b, c)

37. (a, b, c, d) : Here $f(x) = 2x - \sin x$ and $g(x) = \sqrt[3]{x}$

Now $gof(x) = g(f(x))$

$gof = \sqrt[3]{2x - \sin x}$

Clearly, the range of gof is R

Now $\frac{d}{dx}(gof) = \frac{1}{3}\frac{(2-\cos x)}{(2x-\sin x)^{2/3}} > 0 \ \forall \ x \in R$

$\Rightarrow$ $gof(x)$ is strictly increasing function and hence it is one-one function.

$\Rightarrow$ Both $f(x)$ and $g(f)$ are one-one functions.

Since $f(x) \to \pm\infty$ and $g(x) \to \pm\infty$ as $x \to \pm\infty$

Hence they are also onto.

Thus both f and g are one-one and onto.

38. (c,d) : $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \ \therefore \ A^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$$

$\therefore \quad A^2 = -I$

$B = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \Rightarrow B^2 = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$$

$\therefore \quad B^2 = -I$

39. (a) : Here $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t$

$$= \begin{vmatrix} \lambda^2 + 3\lambda & \lambda - 1 & \lambda - 3 \\ \lambda - 1 & -2\lambda & \lambda - 4 \\ \lambda - 3 & \lambda + 3 & 3\lambda \end{vmatrix}$$

Putting $\lambda = 0$ in the above identity, we get

$$t = \begin{vmatrix} 0 & -1 & -3 \\ -1 & 0 & -4 \\ -3 & 3 & 0 \end{vmatrix}$$

$= 1(0 - 12) - 3(-3) = -12 + 9 = -3$

40. (a, b, c, d) : $10! = 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

$= (2 \times 4 \times 6 \times 8 \times 10) \times 3 \times 5 \times 7 \times 9$

$= 2^5(1 \times 2 \times 3 \times 4 \times 5) \times 3 \times 5 \times 7 \times 9$

$= 2^8 \times 3^4 \times 5^2 \times 7^1$

$\therefore \quad p = 8, q = 4, r = 2, s = 1$

$\therefore \quad p = 8 = 2 \times 4 = 2q$, Thus option (a) is correct.

Now $pqrs = 8 \times 4 \times 2 \times 1 = 64$, option (b) is correct.

Again $10! = 2^8 \times 3^4 \times 5^2 \times 7$

$= 2^5 (2^2 \times 7 \times 2 \times 5) \times 3^4 \times 5$

$= 2^5 \times 280 \times 3^4 \times 5$

$\therefore \quad$ 10! is divisible by 280. Thus option (c) is correct.

The product of two integers in 10! is 135 ($15 \times 9 = 135$).

Thus option (d) is correct.

ACE YOUR WAY CBSE XI

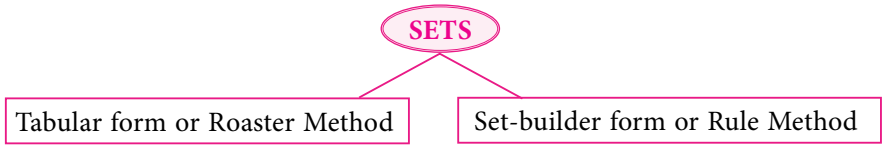
SETS

A set is a well-defined collection of objects. Sets are usually denoted by capital letters A, B, C, etc. and their elements by small letters a, b, c, etc.

Let A be any set of objects and let 'a' be a member of A, then we write $a \in A$ and read it as **'a belongs to A'** or 'a is an element of A' or 'a is a member of A'. If a is not an object of A, then we write $a \notin A$ and read it as **'a does not belong to A'** or 'a is not an element of A' or a' is not a member of A'.

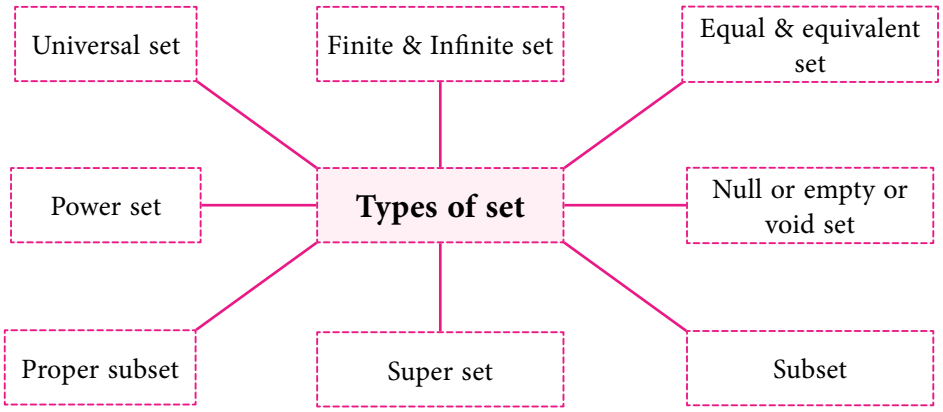
REPRESENTATION OF SETS

There are two ways of expressing a set. These are:



Types	Definition	Example
Tabular form or Roaster Method	In this method we list all the members of the set separating them by commas and enclosing them in curly brackets {}.	Let A be the set consisting of the numbers 1, 3, 4 and 5, then we write $A = \{1, 3, 4, 5\}$.
Set-builder form or Rule Method	In this method, instead of listing all elements of a set, we write the set by some special property or properties satisfied by all its elements and write it as $A = \{x \mid x$ has the property $P(x)\}$ and read it as 'A is the set of all elements x such that x has the property P'. The symbol ' : ' or ' | ' stands for 'such that'.	Let A be the set consisting of the elements 2, 3, 4, 5, 6, 7, 8, 9, 10. Then the set A can be written as : $A = \{x : 2 \le x \le 10$ and $x \in N\}$

TYPES OF SET



- **Finite and Infinite Set :** A set whose elements can be counted *i.e.* a set which has only finite number of elements, is called a finite set and which has an infinite number of elements is called an infinite set.
- **Equal and Equivalent set :** Two sets A and B are called equal if A and B have identical elements. If two sets A and B have equal number of elements then A and B are called equivalent sets.
  Note: Equal sets are always equivalent but equivalent sets may or may not be equal.
- **Null or empty or void set :** A set having no element in it is called a null set. We denote it by ϕ or $\{\ \}$.
- **Subset :** A set A is said to be subset of set B if every member of set A is also the member of set B. It is denoted as $A \subseteq B$. If A is not a subset of B, we write $A \not\subset B$.
  Note :
  (i) Empty set is the subset of every set.
  (ii) Every set is the subset of itself.
  (iii) Total number of subsets of a set having n elements is 2^n.
- **Superset :** If $A \subseteq B$, then B is called superset of A, and we write, $B \supseteq A$.
- **Proper Subset :** If $A \subset B$ and $A \neq B$, then A is called proper subset of B and we write $A \subset B$.
- **Power Set :** Set of all the subsets of a set is called power set of a set which is denoted by $P(A)$. *i.e.*, $P(A) = \{S : S \subset A\}$, where S is subset of set A.
- **Universal Set :** If all sets under consideration are subsets of a larger set, then this larger set is called universal set, denoted by U.

COMPLEMENT OF A SET

- The complement of a given set is a set which contains all those members of the universal set that do not belong to the given set.
- The complement of the set A is denoted by A' or by $A^c = \{x : x \in U, x \notin A\}$.

OPERATIONS ON SETS

Union of Sets

- Let A and B be two given sets. Then the union of A and B is the set of all those elements which belong to either A or B or both.
- The union of A and B is denoted by $A \cup B$ and is read as 'A union B'. The symbol $\cup$ stands for union. Symbolically, $A \cup B = \{x : \text{either } x \in A \text{ or } x \in B\}$

Intersection of Sets

- Let A and B be two given sets. Then the intersection of A and B is the set of elements which belong to both A and B. In other words, the intersection of A and B is the set of common members of A and B.
- The intersection of A and B is denoted by $A \cap B$ and is read as 'A intersection B'. The symbol $\cap$ stands for intersection. Symbolically, $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Disjoint Sets

- If $A \cap B = \phi$, then A and B are said to be disjoint sets.

Difference of two Sets

- Let A and B be two given sets. The difference of sets A and B is the set of elements which are in A but not in B. It is written as $A - B$ and read as 'A difference B'. Symbolically,
  $$A - B = \{x : x \in A \text{ and } x \notin B\}$$
  Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$

Symmetric Difference of Two sets

- Let A and B be two given sets. The symmetric difference of sets A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$. Symbolically, $A \Delta B = \{x : x \notin A \cap B\}$

IMPORTANT PROPERTIES

- **Properties of Union**
  (i) $A \cup B = B \cup A$ (Commutative law)
  (ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)
  (iii) $A \cup \phi = A$ (Law of identity element)
  (iv) $A \cup A = A$ (Idempotent law)
  (v) $U \cup A = U$ (Law of U)
- **Properties of Intersection**
  (i) $A \cap B = B \cap A$ (Commutative law)
  (ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)
  (iii) $\phi \cap A = \phi$, $U \cap A = A$ (Law of ϕ and U)
  (iv) $A \cap A = A$ (Idempotent law)
  (v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law) *i.e.*, $\cap$ distributes over $\cup$.
- **Complement laws**
  (i) $A \cup A' = U$

(ii) $A \cap A' = \phi$

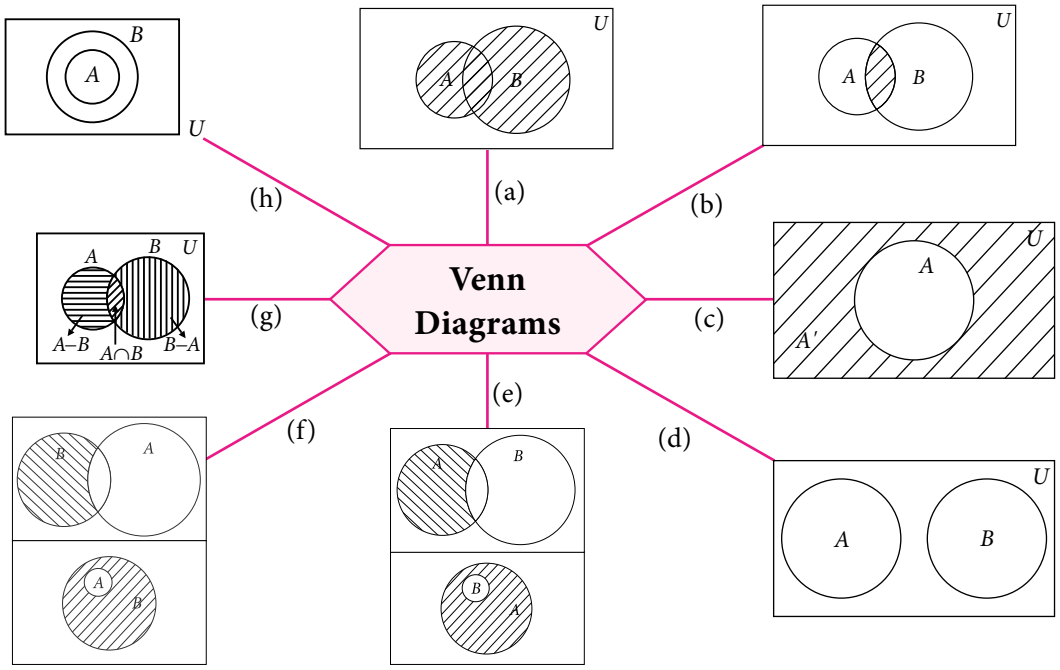
- **De Morgan's law**
  (i) $(A \cup B)' = A' \cap B'$
  (ii) $(A \cap B)' = A' \cup B'$
- **Law of double complementation**
  $(A')' = A$
- **Laws of empty set and universal set**
  $\phi' = U$ and $U' = \phi$

APPLICATION OF SETS

- Let A and B be finite sets and $A \cap B = \phi$ then $n(A \cup B) = n(A) + n(B)$
- If A and B are finite sets such that $A \cap B \neq \phi$ then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- If A, B and C are three finite sets, then $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
- $n(A - B) = n(A) - n(A \cap B)$
- $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

VENN DIAGRAMS

- In Venn diagrams, the universal set U is represented by using rectangle and its subsets are represented by using circle (a closed curve).

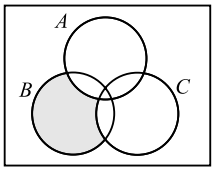


A & B are two sets contained in universal set U.

(a) Shaded area represents $A \cup B$
(b) Shaded area represents $A \cap B$
(c) Shaded area represents A'
(d) No shaded area, *i.e.*, A & B are disjoint sets
(e) Shaded area represents $A - B$
(f) Shaded area represents $B - A$
(g) Shaded area ≡ represents $A - B$; Shaded area |||| represents $B - A$; Shaded area ▨ represents $A \cap B$
(h) This venn diagram shows $A \subseteq B$

Very Short Answer Type

1. If P is the set of all prime numbers and E is the set of all even numbers; find $P \cap E$.
2. If S and T are two sets such that S has 21 elements, T as 32 elements, and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?
3. A is the set of letters in word moon. Find the power set of A.
4. Show by an example, $A \cap B = A \cap C$ need not imply $B = C$.
5. Prove that the shaded region in the given figure is $B - (A \cup C)$.



Short Answer Type

6. Let $A = \{x : x \in R, |x| < 2\}$, $B = \{x : x \in R, |x - 2| \geq 2\}$ and $A \cup B = R - C$, then find the set C.

7. If $U = \{x : x \in N \text{ and } 2 \leq x \leq 12\}$, $A = \{x : x \text{ is an even prime}\}$, $B = \{x : x \text{ is a factor of } 24\}$, then prove that $A - B \neq B \cap A'$.

8. In a class of 175 students, the following data shows the number of students opting one or more subjects.
Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; Mathematics, Physics and Chemistry 18.
Then find the number of students who opted Mathematics alone.

9. Show that for any sets A and B,
$A = (A \cap B) \cup (A - B)$ and $A \cup (B - A) = A \cup B$.

10. Let B be a subset of a set A and
let $P(A : B) = \{X \in P(A) \mid X \supset B\}$
(i) Show that $P(A : \phi) = P(A)$.
(ii) If $A = \{a, b, c, d\}$ and $B = \{a, b\}$. List all the members of the set $P(A : B)$.

Long Answer Type

11. In a certain town, 25% families own a phone and 15% own a car, 65% families own neither a phone nor a car, 2000 families own both a car and a phone.
Consider the following statements in this regard.
I. 10% families own both a car and a phone.
II. 35% families own either a car or a phone.
III. 40000 families live in the town.
Then prove that only II and III are correct.

12. Let A and B be sets, if $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, prove that $A = B$. Also, prove that $(A - B) \cap (B - A) = \phi$.

13. If $U = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $A = \{2, 4, 7\}$, $B = \{3, 5, 7, 9, 11\}$ and $C = \{7, 8, 9, 10, 11\}$, compute
(i) $(A \cap U) \cap (B \cup C)$;
(ii) $C - B$;
(iii) $B - C$;
(iv) $(B - C)'$
(v) U'
(vi) ϕ'

14. If $X = \{8^n - 7n - 1 \mid n \in N\}$ and $Y = \{49n - 49 \mid n \in N\}$, then prove that $X = Y$.

15. On its annual sports day, School awarded 35 medals in athletics, 15 Judo and 18 in swimming. If these medals goes to a total of 58 students and only three of them got medals in all the three sports. Find the number of students who received medals in exactly two of the three sports.

SOLUTIONS

1. P = Set of all prime numbers
$= \{2, 3, 5, 7, 11, ...\}$
E = Set of all even numbers
$= \{2, 4, 6, 8, 10, ...\}$
$\therefore$ $P \cap E = \{2, 3, 5, 7, 11, ...\} \cap \{2, 4, 6, 8, 10, ...\}$
$= \{2\}$

2. $n(S) = 21$, $n(T) = 32$, $n(S \cap T) = 11$, $n(S \cup T) = ?$
$n(S \cup T) = n(S) + n(T) - n(S \cap T)$
$= 21 + 32 - 11 = 42$
Hence, $S \cup T$ has 42 elements.

3. Power set of $A = P(A) = \{\phi, \{m\}, \{o\}, \{n\}, \{m, o\}, \{m, n\}$ $\{o, n\}, \{m, o, n\}\}$

4. Let $A = \{1, 2\}$, $B = \{1, 3\}$ and $C = \{1, 4\}$
Now, $A \cap B = \{1, 2\} \cap \{1, 3\} = \{1\}$
and $A \cap C = \{1, 2\} \cap \{1, 4\} = \{1\}$
$\therefore$ $A \cap B = A \cap C$ but $B \neq C$.

5. In the venn diagram of $B - (A \cup C)$, whole set B is shaded except set $A \cup C$.

6. Given that $A = \{x : x \in R, |x| < 2\}$
and $B = \{x : x \in R, |x - 2| \geq 2\}$
$\Rightarrow$ $A = -2 < x < 2$
and $B = \{x : x \in R, x - 2 \leq -2 \text{ or } x - 2 \geq 2\}$
$= \{x : x \in R, x \leq 0 \text{ or } x \geq 4\}$
$\Rightarrow$ $A =]-2, 2[$ & $B = (-\infty, 0] \cup [4, \infty)$
Now $A \cup B = (-\infty, 2) \cup [4, \infty) = R - [2, 4)$
$\therefore$ $C = [2, 4)$ *i.e.* $2 \leq x < 4$

7. $U = \{x : x \in N \text{ and } 2 \leq x \leq 12\}$
$\Rightarrow$ $U = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$A = \{x : x \text{ is an even prime}\}$ $\Rightarrow$ $A = \{2\}$
$B = \{x : x \text{ is a factor of } 24\}$
$\Rightarrow$ $B = \{2, 3, 4, 6, 8, 12\}$
$\Rightarrow$ $A - B = \{2\} - \{2, 3, 4, 6, 8, 12\} = \phi$
Hence, $A - B$ is an empty set, is a true statement.
Now $A' = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$,

$B \cap A' = \{3, 4, 6, 8, 12\}$

Hence $A - B \neq B \cap A'$

8. Given, $n(M) = 100$, $n(P) = 70$, $n(C) = 40$

$n(M \cap P) = 30$, $n(M \cap C) = 28$

$n(P \cap C) = 23$

and $n(M \cap P \cap C) = 18$

$\therefore \quad n(M \cap P' \cap C') = n[M \cap (P \cup C)']$

(De-Morgan's Law)

$= n(M) - n[M \cap (P \cup C)]$

$= n(M) - n[(M \cap P) \cup (M \cap C)]$

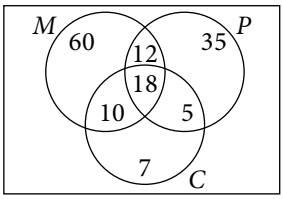
$[\because n(A \cap B') = n(A) - n(A \cap B)]$

$= n(M) - [n(M \cap P) + n(M \cap C)$

$-n(M \cap P \cap C)]$

$= 100 - [30 + 28 - 18]$

$= 100 - 40 = 60$



9. $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$

$[\because A - B = A \cap B']$

$= A \cap (B \cup B')$ [Distributive law]

$= A \cap U$, where U is universal set

$= A$

$A \cup (B - A) = A \cup (B \cap A')$

$[\because B - A = B \cap A']$

$= (A \cup B) \cap (A \cup A')$ [Distributive law]

$= (A \cup B) \cap U$, where $U = A \cup A'$ is universal set

$= A \cup B$ $(\because A \cup B \subset U)$

10. (i) We have,

$P(A : B) = \{X \in P(A) \mid B \subset X\}$

= Set of all those subsets of A which contain B

$\therefore \quad P(A : \phi)$ = Set of all those subsets of A which contain ϕ

= Set of all subsets of set A

$= P(A)$

(ii) We have,

$A = \{a, b, c, d\}$ and $B = \{a, b\}$

$\therefore \quad P(A : B)$ = Set of all those subsets of set A which contain B

$= \{\{a, b, c\}, \{a, b, d\}, \{a, b, c, d\}\}$

$\therefore$ Elements of $P(A : B)$ are $\{a, b, c\}$, $\{a, b, d\}$ and $\{a, b, c, d\}$

11. Let there be x families in the town. Let P and C denote the set of families using phone and car, respectively. Then,

$$n(P) = \frac{25x}{100} = \frac{x}{4}, \; n(C) = \frac{15x}{100} = \frac{3x}{20}$$

and $n(\bar{P} \cap \bar{C}) = \dfrac{65x}{100} = \dfrac{13x}{20}$

Also, $n(P \cap C) = 2000$

Now, $n(\bar{P} \cap \bar{C}) = \dfrac{13x}{20}$

$$\Rightarrow \quad n(\overline{P \cup C}) = \frac{13x}{20}$$

$$\Rightarrow \quad n(U) - n(P \cup C) = \frac{13x}{20}$$

$$\Rightarrow \quad x - \{n(P) + n(C) - n(P \cap C)\} = \frac{13x}{20}$$

$$\Rightarrow \quad x - \left(\frac{x}{4} + \frac{3x}{20} - 2000\right) = \frac{13x}{20}$$

$$\Rightarrow \quad \frac{x}{20} = 2000$$

$\Rightarrow \quad x = 40000$

Thus, Statement III is correct.

10% of the total families in the town is 4000 and it is given that 2000 families own both a car and a phone. So, Statement I is not correct.

Now, $n(P \cup C) = n(P) + n(C) - n(P \cap C)$

$$\Rightarrow \quad n(P \cup C) = \frac{x}{4} + \frac{3x}{20} - 2000$$

$\Rightarrow \quad n(P \cup C) = 10000 + 6000 - 2000 = 14000$

$\Rightarrow \quad n(P \cup C) = 35\%$ of 40000

Thus, Statement II is correct.

12. We have for some set X,

$A \cup X = B \cup X \Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow A = (A \cap B) \cup (A \cap X)$

$[\because \quad A \cap (A \cup X) = A]$

$\Rightarrow A = (A \cap B) \cup \phi$

$[\because A \cap X = \phi$ (given)]

$\Rightarrow A = A \cap B$

$\Rightarrow A \subseteq B$ ...(1)

Again,

$A \cup X = B \cup X \Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow (B \cap A) \cup (B \cap X) = B$

$[\because B \cap (B \cup X) = B]$

$\Rightarrow (B \cap A) \cup \phi = B$

$\Rightarrow B \cap A = B$ $[\because B \cap X = \phi$ (given)]

$\Rightarrow B \subseteq A$ ...(2)

From (1) and (2), we get $A = B$.

Let $x \in A - B \Rightarrow x \in A$ and $x \notin B \Rightarrow x \notin B - A$

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Further any $x \in B - A \Rightarrow x \in B$ and $x \notin A$
$\Rightarrow x \notin A - B$
This proves that $(A - B) \cap (B - A) = \phi$.

13. (i) Here $A \cap U = \{2, 4, 7\}$;
$B \cup C = \{3, 5, 7, 8, 9, 10, 11\}$.
$(A \cap U) \cap (B \cup C) = \{2, 4, 7\} \cap \{3, 5, 7, 8, 9, 10, 11\} = \{7\}$
(ii) $C - B$ is a set of members which belong to C, but do not belong to B.
$\therefore \quad C - B = \{8, 10\}$
(iii) $B - C$ is a set of members which belong to B, but do not belong to C.
$\therefore \quad B - C = \{3, 5\}$
(iv) From (iii),
$B - C = \{3, 5\}$
$\therefore \quad (B - C)' = \{2, 4, 6, 7, 8, 9, 10, 11\}$
(v) U' = Set of members in U which are not in U
$= \phi$, since there is no such member
(vi) $\phi = \{\}$, the empty set,
ϕ' = Set of members in U which are not in $\phi = U$

14. Given, $X = \{8^n - 7n - 1 : n \in N\}$ and
$Y = \{49n - 49 : n \in N\}$
Now, $8^n - 7n - 1 = (7+ 1)^n - 7n - 1$
$= 7^n + {}^nC_1 7^{n-1} + {}^nC_2 7^{n-2} + \ldots + {}^nC_{n-2} 7^2 + {}^nC_{n-1} 7 + {}^nC_n - 7n - 1$
[$\because$ by binomial expansion, $(x + 1)^n = x^n + {}^nC_1 x^{n-1} + {}^nC_2 x^{n-2} + \ldots + {}^nC_{n-2} x^2 + {}^nC_{n-1} x + {}^nC_n$]
$= {}^nC_n 7^n + {}^nC_1 7^{n-1} + {}^nC_2 7^{n-2} + \ldots + {}^nC_{n-2} 7^2 + 7n + 1 - 7n - 1$
$(\because \quad {}^nC_n = {}^nC_0 = 1, {}^nC_{n-1} = {}^nC_1 = n)$
$= {}^nC_n 7^n + {}^nC_1 7^{n-1} + {}^nC_2 7^{n-2} + \ldots + {}^nC_2 7^2$
$(\because \quad {}^nC_{n-2} = {}^nC_2)$
$= {}^nC_2 7^2 + \ldots + {}^nC_2 7^{n-2} + {}^nC_1 7^{n-1} + {}^nC_n 7^n$
$= 7^2[{}^nC_2 + \ldots + {}^nC_n 7^{n-2}]$
$= 49[{}^nC_2 + \ldots + {}^nC_n 7^{n-2}]$
$\therefore \quad 8^n - 7n - 1$ is a multiple of 49 for $n \geq 2$
For $n = 1$, $8^n - 7n - 1 = 8 - 8 = 0$
$\therefore \quad 8^n - 7n - 1$ is a multiple of 49 for all $n \in N$.
X contains elements which are multiples of 49 and clearly Y contains all multiples of 49.
$\therefore \quad X = Y$.

15. Let A denote the set of students who received medal in athletics, J be the set of students who got medal in Judo and S be the set of students who got medal in swimming.
$\because \quad n(A) = 35, n(J) = 15, n(S) = 18$
$n(A \cup J \cup S) = 58$ and $n(A \cap J \cap S) = 3$
Now $n(A \cup J \cup S)$
$= n(A) + n(J) + n(S) - n(A \cap J) - n(A \cap S) - n(J \cap S) + n(A \cap J \cap S)$
$58 = 35 + 15 + 18 + 3 - n(A \cap J) - n(A \cap S) - n(J \cap S)$
$\therefore \quad n(A \cap S) + n(A \cap J) + n(J \cap S) = 71 - 58 = 13$
$\therefore$ Number of students who received medals in exactly two of the three
$= n(A \cap J) + n(J \cap S) + n(S \cap A) - 3n(A \cap S \cap J)$
$= 13 - 3 \times 3 = 13 - 9 = 4$.

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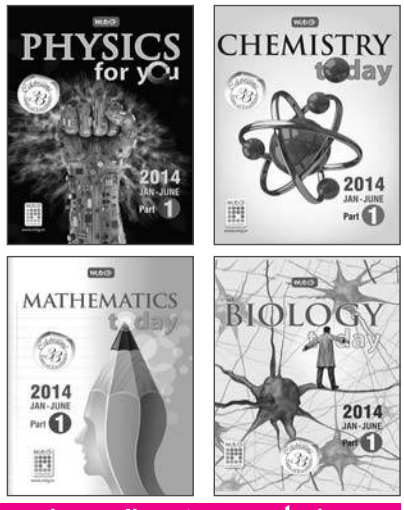

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ACE YOUR WAY CBSE XII

Series 1

RELATIONS AND FUNCTIONS

RELATIONS

Let A and B be two non-empty sets. The relation R between A and B is a subset of $A \times B$. Symbolically, we write the relation between A and B as: $R : A \to B$ if and only if $R \subseteq A \times B$

Domain and Range of a Relation

Domain	If $R : A \to B$, then the domain of R is the set of all first elements of the ordered pairs (x, y) which belong to R.	Symbolically, Domain = $\{x : (x, y) \in R, x \in A$ for some $y \in B\}$
Range	The range of a relation is the set of all second elements of the ordered pairs which belong to R.	Symbolically, Range = $\{y : (x, y) \in R, y \in B$ for some $x \in A\}$

Relation in a set

Let R be a relation from A to B. If $A = B$, then we say that R is a relation in A or R is a relation on A.

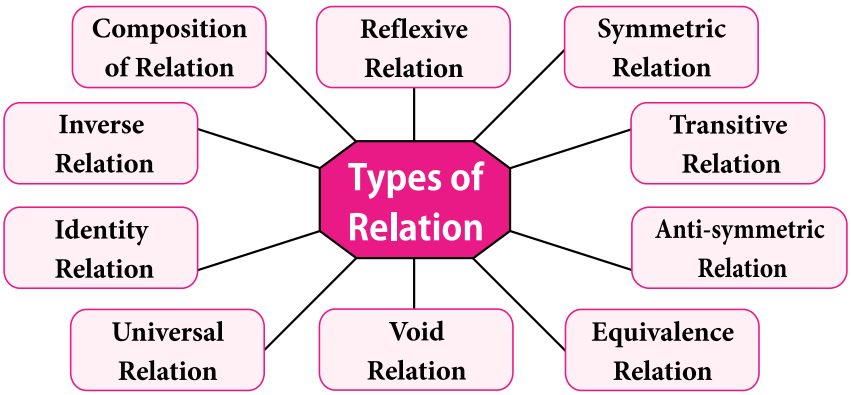
Thus, a relation R in a set A is a subset of the cartesian product $A \times A$.

Total Number of Relations

Let A and B be two non-empty finite sets having n and m elements respectively,

$\therefore$ Total number of relations from A to $B = 2^{nm}$

TYPES OF RELATION



Name	Definition
Reflexive Relation	A relation R on a set A is reflexive if $(a, a) \in R, \ \forall \ a \in A$
Symmetric Relation	A relation R on a set A is symmetric if $(a_1, a_2) \in R \Rightarrow (a_2, a_1) \in R \ \ \forall \ a_1, a_2 \in A$
Transitive Relation	A relation R on a set A is transitive if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R \ \ \forall \ a_1, a_2, a_3 \in A$.
Anti-symmetric Relation	A relation R on a set A is called an anti-symmetric relation, if $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$. If $a \neq b$, then a may be related to b or b may be related to a, but never both.
Equivalence Relation	A relation R is an equivalence relation if R is reflexive, symmetric and transitive.
Void Relation	A relation R on a set A is called an empty or void relation, if no element of A is related to any element of A.
Universal Relation	A relation R on a set A is called universal relation, if each element of A is related to every element of A.
Identity Relation	The relation I_A on set A is identity relation, if every element of A is related to itself only.
Inverse Relation	If R is a relation on set A, then R^{-1} on A is defined as $R^{-1} = \{(b, a) : (a, b) \in R\}$
Composition of Relations	Let $R \subseteq A \times B, S \subseteq B \times C$ be two relations, then composite relation of R and S is $SoR \subseteq A \times C$ or $SoR = \{(a, c); (a, b) \in R, (b, c) \in S\}$

FUNCTIONS

Let A and B be two non-empty sets. If there exists a rule of correspondence by which each element $x \in A$ is related to a unique element $y \in B$, then such correspondence is called the function from A to B. it is written as:

$f : A \to B$ such that $x \in A$ and $y \in B$, where $y = f(x)$

The element $y \in B$ is called the image of x under f and x is called the pre-image of y under f.

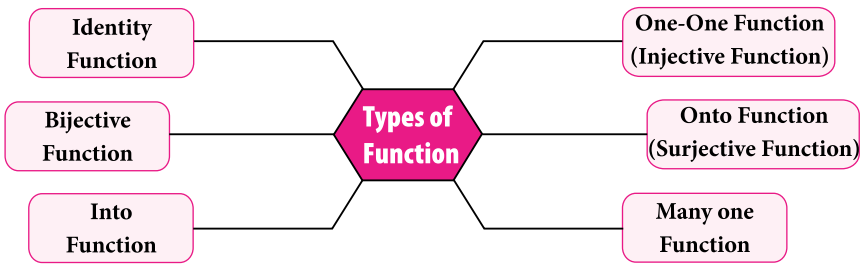
- The number of functions from a finite set A into set $B = [n(B)]^{[n(A)]}$
- There may exist some elements in set B which are not the images of any element in set A.

The element $y \in B$ that is associated to x under f is denoted by $f(x)$ and is called the **value of f at x.**

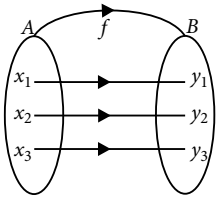
Domain, Co-domain, Range of Function $f(x)$

Domain	The set A is called the domain of function f.
Co-domain	The set B is called the co-domain of function f.
Range	The set $\{f(x) : x \in A$ and $f(x) \in B\}$ for all values taken by f is called the range of f. Obviously, it is a subset of set B.

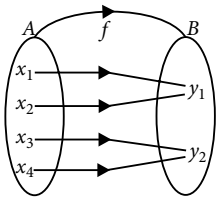
TYPES OF FUNCTIONS



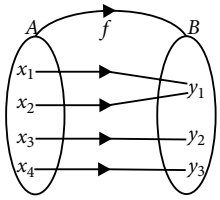
- **One-one function (Injective function) :** A function $f : A \to B$ is defined to be one-one if the images of distinct elements of A under f are distinct in B.



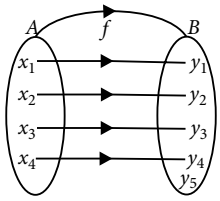
- **Onto function (Surjective function) :**
  Let $f : A \to B$. If every element in B has at least one pre-image in A, then f is said to be an onto function or surjective function.
- If the range of the function equals to the codomain of the function, the function is onto.



- **Many one function :** Let $f : A \to B$. If two or more than two elements of set A have the same image in B, then f is said to be many-one.
- If $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$, for every $x_1, x_2 \in$ domain, then f is one-one or else many one.
- If $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$, for every $x_1, x_2 \in$ domain, then f is one-one or many one.



- **Into function :** Let $f : A \to B$. If there exists a single element in B having no pre-image in A, then f is said to be an into function.



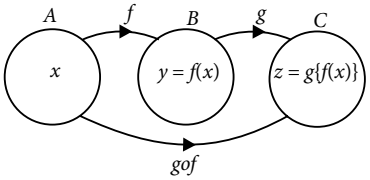
- **Bijective function :** If a function is one-one and onto, then it is bijective.

  or

  A function $f : A \to B$ is bijective if
  (a) f is one-one, *i.e.*, $f(x) = f(y)$
  $\Rightarrow x = y \quad \forall \quad x, y \in A$
  (b) f is onto *i.e.*, $\forall \; y \in B, \exists \; x \in A$ such that $f(x) = y$
- **Identity function :** Identity function I_A on a non-empty set A defined by $I_A : A \to A$, $I_A(x) = x, \quad \forall \; x \in A$.
  This is clearly a one-one function with domain A and range A.

COMPOSITION OF FUNCTIONS AND INVERTIBLE FUNCTION

- **Inverse function :** Inverse of a function $f : A \to B$ defined by $f^{-1} : B \to A$
  $\therefore \quad f^{-1}(y) = x \Leftrightarrow f(x) = y$
- A function f is invertible if and only if f is one-one onto.
- $fog \neq gof$, $fo(goh) = (fog)oh$, If $f : A \to B$, then $foI_A = I_B \; of = f$



- **Composition of functions :** Let $f : A \to B$ and $g : B \to C$ be two functions of non empty sets A, B, C, then $gof : A \to C$ is called composition of f and g defined as $gof(x) = g\{f(x)\} \; \forall \; x \in A$

BINARY OPERATION

Definition : Let A be non-empty set, then the function from $A \times A$ into A is called a binary operation. Symbolically, a function '$*$' which is

$* : A \times A \to A$

is called a binary operation. The image of any $(a, b) \in A \times A$ under '$*$' is denoted by $a * b$.

(i) A binary operation '$*$' over a set A is said to be **commutative** if $a * b = b * a \; \forall \; a, b \in A$

(ii) A binary operation '$*$' over a set A is said to be **associative** if
$(a * b) * c = a * (b * c) \; \forall \; a, b, c \in A$

(iii) Let $*$ be a binary operation on a set A. An element $e \in A$ is said to be an **identity element** for the binary operation $*$ if $a * e = a = e * a \; \forall \; a \in A$ *i.e.*, e when combined with any element $a \in A$ produces a itself.

(iv) Let $*$ be a binary operation on a set A and let e be the identity element of the set A for this operation $*$. An element $b \in A$ is said to be the **inverse of an element** $a \in A$ if $a * b = e = b * a$.

Very Short Answer Type

1. The relation R is defined on the set of natural numbers as $\{(a, b) : a = 2b\}$. Then, find R^{-1}.

2. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R\beta \Leftrightarrow \alpha \perp \beta$, $\alpha, \beta \in L$. Then, prove that R is symmetric.

3. Find the domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

4. Let $f : A \to B$ and $g : B \to C$ be two functions. If both f and g are one-to-one, show that gof is also one-to-one function.

5. Prove that the function $f : N \to N$ given by $f(x) = 2x$ is injective but not surjective.

Short Answer Type

6. Consider the set $N \times N$, the set of ordered pairs of natural numbers. Let R be the relation in $N \times N$ which is defined by $(a, b)\ R(c, d)$ if and only if $a + d = b + c$. Prove that R is an equivalence relation.

7. Show that $f : N \to N$ defined by $f(x) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ is many-one onto function.

8. In the set N of natural numbers define the binary operation $*$ by $m * n$ = L.C.M (m, n); $m, n \in N$
Is the operation commutative and associative?

9. If $f(x) = (ax^2 - b)^3$, then prove that the function g such that $f\{g(x)\} = g\{f(x)\}$ is given by $g(x) = \left(\frac{x^{1/3} + b}{a}\right)^{1/2}$

10. Find the range of
$f(x) = \sqrt{(1 - \cos x)\sqrt{(1 - \cos x)\sqrt{(1 - \cos x)\sqrt{....\infty}}}}$

Long Answer Type

11. Prove that the function $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1 + |x|}$, $x \in R$ is both one-one and onto.

12. Consider $f : R \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6} - 1}{3}\right)$.

13. Let $*$ be a binary operation on the set $R - \{1\}$ of all real numbers defined by the rule $a * b = a + b - ab$ for all $a, b \in R - \{1\}$, whereas on R.H.S. we have usual addition, subtraction and multiplication of real numbers. Show that $*$ is commutative as well as associative. Also find the identity element. What is the inverse of an element a?

14. Show that each of the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by
(i) $R = \{(a, b) : |a - b|$ is a multiple of $4\}$
(ii) $R = \{(a, b) : a = b\}$
is an equivalence relation. Find the set of all elements related to 1 in each case.

15. If $f(x) = \frac{1}{x}$ and $g(x) = \frac{1 - x}{1 + x}$, find D_{gof} and D_{fog}. Also, find $(gof)(x)$ and $(fog)(x)$.

SOLUTIONS

1. $R = \{(2, 1), (4, 2), (6, 3)...\}$
So, $R^{-1} = \{(1, 2), (2, 4), (3, 6), ...\}$

2. Given, $\alpha R\beta \Leftrightarrow \alpha \perp \beta$
Since $\alpha \perp \beta \Leftrightarrow \beta \perp \alpha \Rightarrow \beta R\alpha$
Hence, R is symmetric.

3. $f(x) = g(x) \Rightarrow 2x^2 - 1 = 1 - 3x$

$\Rightarrow (x + 2)(2x - 1) = 0 \Rightarrow x = -2, \frac{1}{2}$
Thus, $f(x)$ and $g(x)$ are equal on the set $\{-2, 1/2\}$.

4. Let $(gof)(x) = (g\ of)(y)$ for $x, y \in A$. Then $g(f(x)) = g(f(y))$. So, $f(x) = f(y)$ as g is one-to-one. Therefore, $x = y$ as f is also one-to-one. Thus, gof is one-to-one.

5. The function f is one-one, as $f(x_1) = f(x_2) \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$. Further, f is not onto, as for $1 \in N$, there does not exist any x in $f(x)$ such that $f(x) = 2x = 1$.

6. **(i) Reflexivity**
$\forall\ (a, b) \in N \times N$, we have
$a + b = b + a$, *i.e.*, $(a, b)\ R\ (a, b)$
$\therefore$ R is reflexive.
(ii) Symmetricity
Suppose $(a, b)\ R\ (c, d)$. Then $a + d = b + c$, which implies that $c + b = d + a$
i.e., $(c, d)\ R\ (a, b)$
Hence R is symmetric.
(iii) Transitivity
Now, suppose $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$
Then $a + d = b + c$ and $c + f = d + e$
Thus, $(a + d) + (c + f) = (b + c) + (d + e)$
Subtracting $c + d$ from both sides, we get
$a + f = b + e$
$\therefore$ $(a, b)\ R\ (e, f)$ and hence R is transitive.
Since R is reflexive, symmetric and transitive, therefore, R is an equivalence relation.

7. We have, $f(1) = \frac{1+1}{2} = 1$ and $f(2) = \frac{2}{2} = 1$.

Thus, $1 \neq 2$ but $f(1) = f(2)$. So, f is a many-one function.

Onto : Let n be an arbitrary element of N.

If n is an odd natural number, then $2n - 1$ is also an odd natural number such that

$$f(2n - 1) = \frac{2n-1+1}{2} = n$$

If n is an even natural number, then $2n$ is also an even number such that

$$f(2n) = \frac{2n}{2} = n.$$

Thus, for every $n \in N$ (whether even or odd), there exists its pre-image in N. So, f is onto.

Hence, f is a many-one onto function.

8. (i) The operation is clearly commutative, since

L.C.M. (m, n) = L.C.M. (n, m)

$\therefore \quad m * n = n * m$

(ii) Again for $l, m, n \in N$,

$m * n$ = L.C.M. (m, n)

and $n * l$ = L.C.M. (n, l)

Now $(m * n) * l$ = L.C.M. $(m * n, l)$

= L.C.M. [L.C.M.(m, n), l]

= L.C.M. [m, L.C.M. (n, l)]

= L.C.M. [$m, n * l$] = $m * (n * l)$

Hence $(m * n) * l = m * (n * l)$

Hence $*$ is associative.

9. $y = f(x) = (ax^2 - b)^3$

$\Rightarrow ax^2 - b = y^{1/3}$

$$\Rightarrow x^2 = \frac{y^{1/3}+b}{a} \Rightarrow x = \left(\frac{y^{1/3}+b}{a}\right)^{1/2}$$

Let $g(x) = \left(\frac{x^{1/3}+b}{a}\right)^{1/2}$

$$\therefore \quad f\{g(x)\} = f\left(\frac{x^{1/3}+b}{a}\right)^{1/2}$$

$$= \left[a\left(\frac{x^{1/3}+b}{a}\right) - b\right]^3 = (x^{1/3})^3 = x$$

and $g\{f(x)\} = g\{(ax^2 - b)^3\}$

$$= \left(\frac{ax^2 - b + b}{a}\right)^{1/2} = (x^2)^{1/2} = x$$

$\Rightarrow g\{f(x)\} = f\{g(x)\}$

Thus, $g(x) = \left(\frac{x^{1/3}+b}{a}\right)^{1/2}$

10. Given

$$f(x) = \sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{...\infty}}}}$$

$$= (1-\cos x)^{\frac{1}{2}} (1-\cos x)^{\frac{1}{4}} (1-\cos x)^{\frac{1}{8}} .....\infty$$

$$= (1-\cos x)^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....\infty} = (1-\cos x)^{\frac{1/2}{1-1/2}} = 1 - \cos x$$

$\Rightarrow$ Range of $f(x)$ is $[0, 2]$

11. It is given that $f : R \to \{x \in R : -1 < x < 1\}$ is defined as $f(x) = \frac{x}{1+|x|}$, $x \in R$.

Suppose $f(x) = f(y)$, where $x, y \in R$

It can be observed that if x is positive and y is negative, then, we have

$$\frac{x}{1+x} = \frac{y}{1-y} \text{ (as } x > 0 \text{ and } y < 0)$$

$$\Rightarrow \quad 2xy = x - y$$

Since x is positive and y is negative, then

$$x > y \Rightarrow x - y > 0$$

But $2xy$ is negative. Then, $2xy \neq x - y$.

Thus, the case of x being positive and y being negative can be ruled out.

Under a similar argument, the case of x being negative and y being positive can also be ruled out. Therefore, x and y have to be either positive or negative. When x and y are both positive, we have

$$f(x) = f(y)$$

$$\Rightarrow \quad \frac{x}{1+x} = \frac{y}{1+y}$$

$$\Rightarrow \quad x + xy = y + xy$$

$$\Rightarrow \quad x = y$$

When x and y are both negative, we have

$$f(x) = f(y)$$

$$\Rightarrow \quad \frac{x}{1-x} = \frac{y}{1-y}$$

$$\Rightarrow \quad x - xy = y - yx$$

$$\Rightarrow \quad x = y$$

Therefore, f is one-one. Now, let $y \in R$ such that $-1 < y < 1$. If y is negative, then there exists $x = \frac{y}{1+y} \in R$ such that

$$f(x) = f\left(\frac{y}{1+y}\right) = \frac{\left(\frac{y}{1+y}\right)}{1+\left|\frac{y}{1+y}\right|} = \frac{\frac{y}{1+y}}{1+\left(\frac{-y}{1+y}\right)}$$

$$= \frac{y}{1+y-y} = y$$

If y is positive, then there exists $x = \frac{y}{1-y} \in R$ such that $f(x) = f\left(\frac{y}{1-y}\right) = \frac{\left(\frac{y}{1-y}\right)}{1+\left|\left(\frac{y}{1-y}\right)\right|} = \frac{\frac{y}{1-y}}{1+\left(\frac{y}{1-y}\right)}$

$$= \frac{y}{1-y+y} = y$$

Also, for $y = 0$, we have $x = 0 \in R$

Therefore, f is onto. Hence, f is both one-one and onto.

12. Let y be an arbitrary element in range of f.

Let $y = 9x^2 + 6x - 5 = 9x^2 + 6x + 1 - 6$

$= (3x + 1)^2 - 6$

$\Rightarrow y + 6 = (3x + 1)^2$ or $3x + 1 = \sqrt{y+6}$

$\therefore \quad x = \frac{\sqrt{y+6}-1}{3} = g(y)$

Let g : Range $f \to R^+$, such that $g(y) = \frac{\sqrt{y+6}-1}{3}$

$gof(x) = g(f(x)) = g(9x^2 + 6x - 5)$

$= g[(3x + 1)^2 - 6]$

$= \frac{\left(\sqrt{(3x+1)^2-6+6}\right)-1}{3} = \frac{3x+1-1}{3} = x$

$\Rightarrow gof(x) = x$

Now, $fog(y) = f(g(y)) = f\left(\frac{\sqrt{y+6}-1}{3}\right)$

$$= \left[3\left(\frac{\sqrt{y+6}-1}{3}\right)+1\right]^2 - 6 = y$$

$\therefore \quad fog(y) = y$

$\Rightarrow gof = I_x, fog = I_y \Rightarrow f$ is invertible.

$\Rightarrow f^{-1}(y) = g(y) = \frac{\sqrt{y+6}-1}{3}$.

13. $a * b = a + b - ab \ \forall \ a, b \in R - \{1\}$ ...(1)(given)

Interchanging a and b in (1),

$b * a = b + a - ba = a + b - ab$ ... (2)

[$\because$ Addition and multiplication are commutative on $R - \{1\}$]

From (1) and (2), we have

$a * b = b * a \ \forall \ a, b \in R - \{1\}$

$\therefore$ $*$ is commutative on $R - \{1\}$.

Associativity : For all $a, b, c \in R - \{1\}$, we have

$(a * b) * c = (a + b - ab) * c$ (By (1))

$= (a + b - ab) + c - (a + b - ab)\, c$ (By (1))

$= a + b - ab + c - ac - bc + abc$

$= a + b + c - ab - bc - ac + abc$ ... (3)

Again $a * (b * c) = a * (b + c - bc)$ (By (1))

$= a + (b + c - bc) - a\,(b + c - bc)$ (By (1))

$= a + b + c - bc - ab - ac + abc$

$= a + b + c - ab - bc - ac + abc$ ... (4)

From (3) and (4), we have

$(a * b) * c = a * (b * c) \ \forall \ a, b, c \in R - \{1\}$

$\therefore$ Binary operation $*$ is associative in $R - \{1\}$.

Identity element : Let $e \in R - \{1\}$ be the identity for binary operation $*$.

$\therefore \quad a * e = a \ \forall \ a \in R - \{1\}$

$\Rightarrow a + e - ae = a$ [Putting $b = e$ in (1)]

or $e - ae = 0$ or $e(1 - a) = 0 \ \forall \ a \in R - \{1\}$

But $1 - a \neq 0$ [$\because a \in R - \{1\} \Rightarrow a \neq 1 \Rightarrow 1 - a \neq 0$]

$\therefore \quad e = 0$ is the identity element of $R - \{1\}$. ... (5)

Inverse of an element : Let $b \in R - \{1\}$ be the inverse of an element $a \in R - \{1\}$.

Then $a * b = e$

$\therefore \quad a + b - ab = 0$ [By (5)]

or $b(1 - a) = -a \quad \therefore \quad b = -\frac{a}{1-a} = \frac{a}{a-1}$

Since $a \in R - \{1\} \Rightarrow a \neq 1 \Rightarrow a - 1 \neq 0$

Again $b \neq 1$ [$\because$ if $b = 1$ then $\frac{a}{a-1} = 1$ or $a = a - 1$ or $0 = -1$ (Impossible)]

$\therefore \quad b = \frac{a}{a-1} \in R - \{1\}$ is the inverse of $a \in R - \{1\}$.

14. The set $A = \{x \in Z : 0 \le x \le 12\} = \{0, 1, 2, ..., 12\}$

(i) $R = \{(a, b) : |a - b|$ is a multiple of $4\}$

$|a - b| = 4k$ or $b = a + 4k$

$\therefore \quad R = \{(1, 5), (1, 9), (2, 6), (2, 10), (3, 7), (3, 11), (4, 8), (4, 12), (5, 9), (6, 10), (7, 11), (8, 12), (0, 0), (1, 1), (2, 2), ..., (12, 12)\}$

(a) $a - a = 0 = 4k$, where $k = 0 \Rightarrow (a, a) \in R$,

$\therefore \quad R$ is reflexive.

(b) If $|a - b| = 4k$, then $|b - a| = 4k$ *i.e.* (a, b) and (b, a) both belong to R $\quad \therefore \ R$ is symmetric.

(c) $a - c = a - b + b - c$

When $a - b$ and $b - c$ both are multiples of 4 then $a - c$ is also a multiple of 4.

This shows if $(a, b), (b, c) \in R$, then $a - c$ also $\in R$. $\therefore$ R is transitive.

$\therefore$ R is an equivalence relation.

Then set related to 1 are $\{(1, 5), (1, 9)\}$

(ii) $R = \{(a, b) : a = b\} = \{(0, 0), (1, 1), (2, 2), ..., (12, 12)\}$

(a) $a = a \Rightarrow (a, a) \in R$ $\therefore$ R is reflexive.

(b) Again if $(a, b) \in R$, then (b, a) also $\in R$

$\Rightarrow$ R is symmetric.

(c) If $(a, b) \in R$ and $(b, c) \in R \Rightarrow a = b = c$

$\therefore$ $a = c \Rightarrow (a, c) \in R$. Hence R is transitive.

Hence, R is an equivalence relation.

Set related to 1 is $\{1\}$.

15. Given $f(x) = \frac{1}{x}$ and $g(x) = \frac{1-x}{1+x}$

Here, $D_f = \{x \in R : x \neq 0\} = R - \{0\}$

and $R_f = \left\{y \in R : y = \frac{1}{x}\right\} = \left\{y \in R : x = \frac{1}{y} \in R - \{0\}\right\}$

$= \{y \in R : y \neq 0\} = R - \{0\}$

Also, $D_g = \{x \in R : 1 + x \neq 0\} = R - \{-1\}$

and $R_g = \left\{y \in R : y = \frac{1-x}{1+x}\right\} = \left\{y \in R : x = \frac{1-y}{1+y} \in R - \{-1\}\right\}$

$= \{y \in R : y \neq -1\} = R - \{-1\}$

gof **:** Here $R_f \not\subset D_g$, but $R_f \cap D_g \neq \phi$

$\therefore$ gof is defined and $D_{gof} = \{x \in D_f : f(x) \in D_g\}$

$$= \left\{x \in R - \{0\} : \frac{1}{x} \in R - \{-1\}\right\}$$

$$= \left\{x \in R : x \neq 0, \frac{1}{x} \neq -1\right\} = R - \{0, -1\}$$

Also, for all $x \in R - \{0, -1\}$,

$$(gof)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1 - \frac{1}{x}}{1 + \frac{1}{x}} = \frac{x-1}{x+1}.$$

fog **:** Here, $R_g \not\subset D_f$, but $R_g \cap D_f \neq \phi$

$\therefore$ fog is defined and $D_{fog} = \{x \in D_g : g(x) \in D_f\}$

$$= \left\{x \in R - \{-1\} : \frac{1-x}{1+x} \in R - \{0\}\right\}$$

$$= \left\{x \in R : x \neq -1, \frac{1-x}{1+x} \neq 0\right\} = R - \{-1, 1\}$$

Also, for all $x \in R - \{-1, 1\}$

$$(fog)(x) = f(g(x)) = f\left(\frac{1-x}{1+x}\right) = \frac{1}{\frac{1-x}{1+x}} = \frac{1+x}{1-x}.$$

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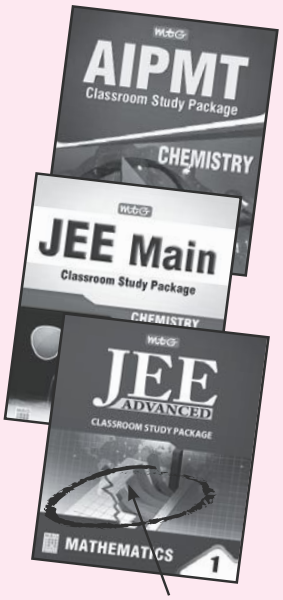

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SOLVED PAPER 2015

Kerala PET

1. The angle between the two vectors $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-2\hat{j}+2\hat{k}$ is equal to

(a) $\cos^{-1}\left(\frac{2}{3}\right)$ (b) $\cos^{-1}\left(\frac{1}{6}\right)$

(c) $\cos^{-1}\left(\frac{5}{6}\right)$ (d) $\cos^{-1}\left(\frac{1}{18}\right)$

(e) $\cos^{-1}\left(\frac{1}{3}\right)$

2. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are coplanar and $|\vec{c}|=\sqrt{3}$, then

(a) $\alpha=\sqrt{2}, \beta=1$ (b) $\alpha = 1, \beta = \pm 1$

(c) $\alpha = \pm 1, \beta = 1$ (d) $\alpha = \pm 1, \beta = -1$

(e) $\alpha = -1, \beta = \pm 1$

3. Let $P(1, 2, 3)$ and $Q(-1, -2, -3)$ be the two points and let O be the origin. Then $|\overrightarrow{PQ}+\overrightarrow{OP}|=$

(a) $\sqrt{13}$ (b) $\sqrt{14}$ (c) $\sqrt{24}$ (d) $\sqrt{12}$

(e) $\sqrt{8}$

4. Let $ABCD$ be a parallelogram. If $\overrightarrow{AB}=\hat{i}+3\hat{j}+7\hat{k}, \overrightarrow{AD}=2\hat{i}+3\hat{j}-5\hat{k}$ and $\vec{p}$ is a unit vector parallel to $\overrightarrow{AC}$, then $\vec{p}$ is equal to

(a) $\frac{1}{3}(2\hat{i}+\hat{j}+2\hat{k})$ (b) $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k})$

(c) $\frac{1}{7}(3\hat{i}+6\hat{j}+2\hat{k})$ (d) $\frac{1}{7}(6\hat{i}+2\hat{j}+3\hat{k})$

(e) $\frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k})$

5. Let $\overrightarrow{OB}=\hat{i}+2\hat{j}+2\hat{k}$ and $\overrightarrow{OA}=4\hat{i}+2\hat{j}+2\hat{k}$. The distance of the point B from straight line passing through A and parallel to the vector $2\hat{i}+3\hat{j}+6\hat{k}$ is

(a) $\frac{7\sqrt{5}}{9}$ (b) $\frac{5\sqrt{7}}{9}$ (c) $\frac{3\sqrt{5}}{7}$ (d) $\frac{9\sqrt{5}}{7}$

(e) $\frac{9\sqrt{7}}{5}$

6. If $\vec{a}=\lambda\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+2\hat{j}+\lambda\hat{k}$ are at right angle, then the value of $|\vec{a}+\vec{b}|-|\vec{a}-\vec{b}|$ is equal to

(a) 2 (b) 1 (c) 0 (d) −1

(e) −2

7. Let the position vectors of the points A, B and C be $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. Let Q be the point of intersection of the medians of the triangle ABC. Then $\overrightarrow{QA}+\overrightarrow{QB}+\overrightarrow{QC}=$

(a) $\frac{\vec{a}+\vec{b}+\vec{c}}{2}$ (b) $2\vec{a}+\vec{b}+\vec{c}$

(c) $\vec{a}+\vec{b}+\vec{c}$ (d) $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

(e) $\vec{0}$

8. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

(e) $\frac{2\pi}{3}$

9. The projection of the line segment joining $(2, 0, -3)$ and $(5, -1, 2)$ on a straight line whose direction ratios are 2, 4, 4 is equal to

(a) $\frac{11}{6}$ (b) $\frac{10}{3}$ (c) $\frac{13}{3}$ (d) $\frac{13}{6}$

(e) $\frac{11}{3}$

10. The angle between the straight line

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + s(\hat{i} - \hat{j} + \hat{k})$ and the plane

$\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$ is

(a) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (b) $\sin^{-1}\left(\frac{\sqrt{2}}{6}\right)$

(c) $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (d) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(e) $\sin^{-1}\left(\frac{2}{3}\right)$

11. If a straight line makes angles α, β, γ with the coordinate axes, then $\frac{1-\tan^2\alpha}{1+\tan^2\alpha} + \frac{1}{\sec 2\beta} - 2\sin^2\gamma =$

(a) –1 (b) 1 (c) –2 (d) 2

(e) 0

12. The equation of the plane which bisects the line segment joining the points (3, 2, 6) and (5, 4, 8) and is perpendicular to the same line segment, is

(a) $x + y + z = 16$ (b) $x + y + z = 10$

(c) $x + y + z = 12$ (d) $x + y + z = 14$

(e) $x + y + z = 15$

13. The foot of the perpendicular from the point (1, 6, 3) to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is

(a) (1, 3, 5) (b) (–1, –1, –1)

(c) (2, 5, 8) (d) (–2, –3, –4)

(e) $\left(\frac{1}{2}, 2, \frac{1}{2}\right)$

14. The plane $x + 3y + 13 = 0$ passes through the line of intersection of the planes $2x - 8y + 4z + p = 0$ and $3x - 5y + 4z - 10 = 0$. If the plane is perpendicular to the plane $3x - y - 2z - 4 = 0$, then the value of p is equal to

(a) 2 (b) 5 (c) 9 (d) 3

(e) –1

15. If a straight line makes the angles 60°, 45° and α with x, y and z axes respectively, then $\sin^2\alpha =$

(a) $\frac{3}{4}$ (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) 1

(e) $\frac{1}{4}$

16. Let A and B be two events. Then $1 + P(A \cap B) - P(B) - P(A)$ is equal to

(a) $P(\bar{A} \cup \bar{B})$ (b) $P(A \cap \bar{B})$

(c) $P(\bar{A} \cap B)$ (d) $P(A \cup B)$

(e) $P(\bar{A} \cap \bar{B})$

17. The mean of five observations is 4 and their variance is 5.2. If three of these observations are 2, 4, and 6, then the other two observations are

(a) 3 and 5 (b) 2 and 6

(c) 4 and 4 (d) 8 and 10

(e) 1 and 7

18. The first term of an A.P. is 148 and the common difference is –2. If the A.M. of first n terms of the A.P. is 125, then the value of n is

(a) 18 (b) 24 (c) 30 (d) 36

(e) 48

19. If the combined mean of two groups is $\frac{40}{3}$ and if the mean of one group with 10 observations is 15, then the mean of the other group with 8 observations is equal to

(a) $\frac{46}{3}$ (b) $\frac{35}{4}$ (c) $\frac{45}{4}$ (d) $\frac{41}{4}$

(e) $\frac{43}{4}$

20. A function f satisfies the relation $f(n^2) = f(n) + 6$ for $n \geq 2$ and $f(2) = 8$. Then the value of $f(256)$ is

(a) 24 (b) 26 (c) 22 (d) 28

(e) 32

21. $\lim_{x\to 0}\left(\frac{10}{9}\frac{\sin 9x}{\sin 10x}\right)\left(\frac{8}{7}\frac{\sin 7x}{\sin 8x}\right)\left(\frac{6}{5}\frac{\sin 5x}{\sin 6x}\right)\left(\frac{4}{3}\frac{\sin 3x}{\sin 4x}\right)\left(\frac{\sin x}{\sin 2x}\right) =$

(a) $\frac{63}{256}$ (b) $\frac{1}{6}$ (c) $\frac{6}{5}$ (d) $\frac{1}{2}$

(e) $\frac{256}{63}$

22. The number of points at which the function $f(x) = \frac{1}{\log_e|x|}$ is discontinuous, is

(a) 1 (b) 2 (c) 3 (d) 4

(e) infinitely many

23. The value of $\lim_{y\to\infty}\left[y\sin\left(\frac{1}{y}\right) - \frac{1}{y}\right]$ is equal to

(a) 1 (b) ∞ (c) –1 (d) 0

(e) $-\infty$

24. $\lim_{x\to\infty}\left(\frac{x^2}{3x-2} - \frac{x}{3}\right) =$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{-2}{3}$ (d) $\frac{-2}{9}$

(e) $\frac{2}{9}$

25. The functions f, g and h satisfy the relations $f'(x) = g(x+1)$ and $g'(x) = h(x-1)$. Then $f''(2x)$ is equal to

(a) $h(2x)$ (b) $4h(2x)$
(c) $h(2x-1)$ (d) $h(2x+1)$
(e) $2h'(2x)$

26. If $f(x) = \sin^{-1}\left(\frac{1-\cos 2x}{2\sin x}\right)$, then $|f'(x)|$ is equal to

(a) $|\sin x|$ (b) x
(c) 0 (d) $|\cos x|$
(e) 1

27. If $y^2 = 100\tan^{-1}x + 45\sec^{-1}x + 100\cot^{-1}x + 45\,\text{cosec}^{-1}x$, then $\frac{dy}{dx} =$

(a) $\frac{x^2-1}{x^2+1}$ (b) $\frac{x^2+1}{x^2-1}$
(c) 1 (d) 0
(e) $\frac{1}{x\sqrt{x^2-1}}$

28. If $f(x) = 3x^2 - 7x + 5$, then $\lim_{x\to 0}\frac{f(x)-f(0)}{x}$ is equal to

(a) 6 (b) −7 (c) 7 (d) −6
(e) 5

29. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{x}{\sqrt{1+x^2}}$ (b) $x\sqrt{1+x^2}$
(c) $\sqrt{1+x^2}$ (d) $\frac{1}{\sqrt{1+x^2}}$
(e) $\frac{x}{1+x}$

30. If $|t|<1$, $\sin x = \frac{2t}{1+t^2}$, $\tan y = \frac{2t}{1-t^2}$, then $\frac{dy}{dx} =$

(a) $\frac{1}{x}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) $-\frac{1}{x}$
(e) 1

31. If $y = \frac{x}{x+1} + \frac{x+1}{x}$, then $\frac{d^2y}{dx^2}$ at $x = 1$ is equal to

(a) $\frac{7}{4}$ (b) $\frac{7}{8}$ (c) $\frac{1}{4}$ (d) $\frac{-7}{8}$
(e) $\frac{-7}{4}$

32. Let $f(x) = (3\sin^2(10x+11) - 7)^2$ for $x \in R$. Then the maximum value of the function f, is

(a) 9 (b) 16 (c) 49 (d) 100
(e) 121

33. If a circular plate is heated uniformly, its area expands $3c$ times as fast as its radius, then the value of c when the radius is 6 units, is

(a) 4π (b) 2π (c) 6π (d) 3π
(e) 8π

34. The slope of the tangent to the curve $y = 3x^2 - 5x + 6$ at (1, 4) is

(a) −2 (b) 1 (c) 0 (d) −1
(e) 2

35. The chord joining the points (5, 5) and (11, 227) on the curve $y = 3x^2 - 11x - 15$ is parallel to tangent at a point on the curve. Then the abscissa of the point is

(a) −4 (b) 4 (c) −8 (d) 8
(e) 6

36. The function $f(x) = \sin x - kx - c$, where k and c are constants, decreases always when

(a) $k > 1$ (b) $k \geq 1$ (c) $k < 1$ (d) $k \leq 1$
(e) $k < -1$

37. If $y = m\log x + nx^2 + x$ has its extreme values at $x = 2$ and $x = 1$, then $2m + 10n =$

(a) −1 (b) −4 (c) −2 (d) 1
(e) −3

38. If $s = 2t^3 - 6t^2 + at + 5$ is the distance travelled by a particle at time t and if the velocity is −3 when its acceleration is zero, then the value of a is

(a) −3 (b) 3 (c) 4 (d) −4
(e) 2

39. $\int \frac{(1+x)e^x}{\cot(xe^x)}dx$ is equal to

(a) $\log|\cos(xe^x)| + C$ (b) $\log|\cot(xe^x)| + C$
(c) $\log|\sec(xe^{-x})| + C$ (d) $\log|\cos(xe^{-x})| + C$
(e) $\log|\sec(xe^x)| + C$

40. $\int \frac{x^5}{\sqrt{1+x^3}}dx =$

(a) $\frac{2}{9}\sqrt{1+x^2}\,(x^3-9)+C$ (b) $\frac{2}{9}\sqrt{x^3-9}\,(1+x^2)+C$
(c) $\frac{2}{9}\sqrt{1+x^3}+C$ (d) $\frac{2}{9}\sqrt{1+x^3}\,(x^3-2)+C$
(e) $\frac{2}{9}\sqrt{1+x^2}\,(x^3+9)+C$

41. $\int \frac{4e^x}{2e^x - 5e^{-x}}\,dx =$

(a) $4\log|e^x - 5| + C$ (b) $\frac{1}{4}\log|e^{2x} - 5| + C$

(c) $\log|2e^x - 5e^{-x}| + C$ (d) $4\log|2e^x - 5| + C$

(e) $\log|2e^{2x} - 5| + C$

42. $\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx =$

(a) $\frac{x^2}{2} + 2x + \log|x| + C$

(b) $\frac{x^2}{2} + 2 + \log|x| + C$

(c) $\frac{x^2}{2} + x + \log|x| + C$

(d) $\frac{x^2}{2} + 2x + 2\log|x| + C$

(e) $\frac{x^2}{2} - 2x + \log|x| + C$

43. $\int \frac{x^{n-1}}{x^{2n} + a^2}\,dx =$

(a) $\frac{1}{na}\tan^{-1}\left(\frac{x^n}{a}\right) + C$ (b) $\frac{n}{a}\tan^{-1}\left(\frac{x^n}{a}\right) + C$

(c) $\frac{n}{a}\sin^{-1}\left(\frac{x^n}{a}\right) + C$ (d) $\frac{n}{a}\cos^{-1}\left(\frac{x^n}{a}\right) + C$

(e) $\frac{1}{na}\cot^{-1}\left(\frac{x^n}{a}\right) + C$

44. $\int \frac{(x+1)^2}{x(x^2+1)}\,dx =$

(a) $\log|x(x^2 + 1)| + C$

(b) $\log|x| + C$

(c) $\log|x| + 2\tan^{-1}x + C$

(d) $\log\left(\frac{1}{1+x^2}\right) + C$

(e) $2\log|x| + \tan^{-1}x + C$

45. $\int \frac{1}{x\log x^2}\,dx =$

(a) $\frac{1}{2}\log|\log x^2| + C$ (b) $\log|\log x^2| + C$

(c) $2\log|\log x^2| + C$ (d) $4\log|\log x^2| + C$

(e) $\frac{1}{4}\log|\log x^2| + C$

46. $\int_0^1 \frac{1}{(x^2+16)(x^2+25)}\,dx =$

(a) $\frac{1}{5}\left[\frac{1}{4}\tan^{-1}\left(\frac{1}{4}\right) - \frac{1}{5}\tan^{-1}\left(\frac{1}{5}\right)\right]$

(b) $\frac{1}{9}\left[\frac{1}{4}\tan^{-1}\left(\frac{1}{4}\right) - \frac{1}{5}\tan^{-1}\left(\frac{1}{5}\right)\right]$

(c) $\frac{1}{4}\left[\frac{1}{4}\tan^{-1}\left(\frac{1}{4}\right) - \frac{1}{5}\tan^{-1}\left(\frac{1}{5}\right)\right]$

(d) $\frac{1}{9}\left[\frac{1}{5}\tan^{-1}\left(\frac{1}{4}\right) - \frac{1}{4}\tan^{-1}\left(\frac{1}{5}\right)\right]$

(e) $\frac{1}{9}\left[\frac{3}{4}\tan^{-1}\left(\frac{1}{4}\right) - \frac{4}{5}\tan^{-1}\left(\frac{1}{5}\right)\right]$

47. $\int_{-1}^{1} x(1-x)(1+x)dx =$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) -1

(e) 0

48. The area bounded by $y = x^2 + 3$ and $y = 2x + 3$ is (in sq. units)

(a) $\frac{12}{7}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{8}{3}$

(e) $\frac{3}{8}$

49. The value of $\int_{-\pi}^{\pi} \frac{\sin^2 x}{1+7^x}\,dx$ is equal to

(a) 7^π (b) π (c) $\frac{\pi}{2}$ (d) 2^π

(e) 7π

50. $\int_0^{\sqrt{\pi}/2} 2x^3\sin(x^2)dx =$

(a) $\frac{1}{\sqrt{2}}\left(1 + \frac{\pi}{4}\right)$ (b) $\frac{1}{\sqrt{2}}\left(1 - \frac{\pi}{4}\right)$

(c) $\frac{1}{\sqrt{2}}\left(\frac{\pi}{2} - 1\right)$ (d) $\frac{1}{\sqrt{2}}\left(1 - \frac{\pi}{2}\right)$

(e) $\frac{1}{\sqrt{2}}\left(\frac{\pi}{4} - 1\right)$

51. If $x^2 + y^2 = 1$, then

(a) $yy'' + (y')^2 + 1 = 0$ (b) $yy'' + 2(y')^2 + 1 = 0$

(c) $yy'' - 2(y')^2 + 1 = 0$ (d) $yy'' + (y')^2 - 1 = 0$

(e) $yy'' - (2y')^2 - 1 = 0$

52. The solution of the differential equation $y'(y^2 - x) = y$ is

(a) $y^3 - 3xy = C$ (b) $y^3 + 3xy = C$
(c) $x^3 - 3xy = C$ (d) $y^3 - xy = C$
(e) $x^3 - xy = C$

53. The order and degree of the differential equation $\left[2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^3y}{dx^3}$ are respectively

(a) 2 and 2 (b) 2 and 1
(c) 3 and 2 (d) 3 and 3
(e) 2 and 4

54. The slope of a curve at any point (x, y) other than the origin, is $y + \frac{y}{x}$. Then the equation of the curve is

(a) $y = Cxe^x$ (b) $y = x(e^x + C)$
(c) $xy = Ce^x$ (d) $y + xe^x = C$
(e) $(y - x)e^x = C$

55. The domain of the function $f(x) = \sqrt{7 - 3x} + \log_e x$ is

(a) $0 < x < \infty$ (b) $\frac{7}{3} \le x < \infty$
(c) $0 < x \le \frac{7}{3}$ (d) $-\infty < x < 0$
(e) $-\infty < x \le \frac{7}{3}$

56. If $f(1) = 1, f(2n) = f(n)$ and $f(2n + 1) = (f(n))^2 - 2$ for $n = 1, 2, 3, ...$, then the value of $f(1) + f(2) + ... + f(25)$ is equal to

(a) 1 (b) –15 (c) –17 (d) –1
(e) 13

57. Let X and Y be two non-empty sets such that $X \cap A = Y \cap A = \phi$ and $X \cup A = Y \cup A$ for some non-empty set A. Then

(a) X is a proper subset of Y
(b) Y is a proper subset of X
(c) $X = Y$
(d) X and Y are disjoint sets
(e) $X|A = \phi$

58. The set $(A|B) \cup (B|A)$ is equal to

(a) $[A|(A \cap B)] \cap [B|(A \cap B)]$
(b) $(A \cup B)|(A \cap B)$ (c) $A|(A \cap B)$
(d) $\overline{(A \cap B)}|(A \cup B)$ (e) $(\bar{A} \cup B)|\overline{(A \cup B)}$

59. The range of the function $f(x) = \frac{1}{2 - \cos 3x}$ is

(a) $(-2, \infty)$ (b) $[-2, 3]$
(c) $\left(\frac{1}{3}, 2\right)$ (d) $\left(\frac{1}{2}, 1\right)$
(e) $\left[\frac{1}{3}, 1\right]$

60. In a class of 80 students numbered 1 to 80, all odd numbered students opt for Cricket, students whose numbers are divisible by 5 opt for Football and those whose numbers are divisible by 7 opt for Hockey. The number of students who do not opt any of the three games, is

(a) 13 (b) 24 (c) 28 (d) 52
(e) 67

61. If $\text{Re}(1 + iy)^3 = -26$, where y is a real number, then the value of $|y|$ is

(a) 2 (b) 3 (c) 4 (d) 6
(e) 9

62. If $z = x + iy$ is a complex number such that $|z| = \text{Re}(iz) + 1$, then the locus of z is

(a) $x^2 + y^2 = 1$ (b) $x^2 = 2y - 1$
(c) $y^2 = 2x - 1$ (d) $y^2 = 1 - 2x$
(e) $x^2 = 1 - 2y$

63. Let $i^2 = -1$. Then

$$\left(i^{10} - \frac{1}{i^{11}}\right) + \left(i^{11} - \frac{1}{i^{12}}\right) + \left(i^{12} - \frac{1}{i^{13}}\right) + \left(i^{13} - \frac{1}{i^{14}}\right) + \left(i^{14} + \frac{1}{i^{15}}\right) =$$

(a) $-1 + i$ (b) $-1 - i$
(c) $1 + i$ (d) $-i$
(e) i

64. If $f(z) = \frac{1 - z^3}{1 - z}$, where $z = x + iy$ with $z \ne 1$, then $\text{Re}\{\overline{f(z)}\} = 0$ reduces to

(a) $x^2 + y^2 + x + 1 = 0$
(b) $x^2 - y^2 + x - 1 = 0$
(c) $x^2 - y^2 - x + 1 = 0$
(d) $x^2 - y^2 + x + 1 = 0$
(e) $x^2 - y^2 + x + 2 = 0$

65. If $z = 1 + i$, then the argument of z^2e^{z-i} is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$
(e) 0

66. If the difference between the roots of $x^2 + 2px + q = 0$ is two times the difference between the roots of $x^2 + qx + \frac{p}{4} = 0$, where $p \ne q$, then

(a) $p - q + 1 = 0$ (b) $p - q - 1 = 0$
(c) $p + q - 1 = 0$ (d) $p + q + 1 = 0$
(e) $q - 4p + 1 = 0$

67. Sum of the roots of the equation $|x - 3|^2 + |x - 3| - 2 = 0$ is equal to

(a) 2 (b) 4 (c) 6 (d) 16

(e) -2

68. The quadratic equation whose roots are three times the roots of the equation $2x^2 + 3x + 5 = 0$, is
(a) $2x^2 + 9x + 45 = 0$ (b) $2x^2 + 9x - 45 = 0$
(c) $5x^2 + 9x + 45 = 0$ (d) $2x^2 - 9x + 45 = 0$
(e) $2x^2 + 9x + 49 = 0$

69. If x is real number, then $\frac{x}{x^2 - 5x + 9}$ must lie between
(a) $\frac{1}{11}$ and 1 (b) -1 and $\frac{1}{11}$
(c) -11 and 1 (d) 1 and 11
(e) $-\frac{1}{11}$ and 1

70. If the roots of the equation $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are equal then $a^2 + b^2 + c^2 =$
(a) $a + b + c$ (b) $2a + b + c$
(c) $3abc$ (d) $ab + bc + ca$
(e) abc

71. If one root of a quadratic equation is $\frac{1}{1+\sqrt{3}}$, then the quadratic equation is
(a) $2x^2 + x - 1 = 0$ (b) $2x^2 - 2x - 1 = 0$
(c) $2x^2 + 2x + 1 = 0$ (d) $2x^2 + x + 1 = 0$
(e) $2x^2 + 2x - 1 = 0$

72. The 5^{th} and 8^{th} terms of a G.P. are 1458 and 54 respectively. The common ratio of the G.P. is
(a) $\frac{1}{3}$ (b) 3 (c) 9 (d) $\frac{1}{9}$
(e) $\frac{1}{8}$

73. Let $S(n)$ denote the sum of the digits of a positive integer n. For example, $S(178) = 1 + 7 + 8 = 16$. Then the value of $S(1) + S(2) + S(3) + ... + S(99)$ is equal to
(a) 476 (b) 998 (c) 782 (d) 900

74. An A.P. consists of 23 terms. If the sum of the 3 terms in the middle is 141 and the sum of the last 3 terms is 261, then the first term is
(a) 6 (b) 5 (c) 4 (d) 3
(e) 2

75. If 4^{th} term of a G.P. is 32 whose common ratio is half of the first term, then the 15^{th} term is
(a) 2^{12} (b) 2^{18} (c) 2^{14} (d) 2^{16}
(e) 2^{10}

76. Let S_1 be a square of side 5 cm. Another square S_2 is drawn by joining the midpoints of the sides of S_1. Square S_3 is drawn by joining the midpoints of the sides of S_2 and so on. Then (area of S_1 + area of S_2 + area of S_3 + ... + area of S_{10}) =
(a) $25\left(1-\frac{1}{2^{10}}\right)$ (b) $50\left(1-\frac{1}{2^{10}}\right)$
(c) $2\left(1-\frac{1}{2^{10}}\right)$ (d) $1-\frac{1}{2^{10}}$
(e) $10\left(1-\frac{1}{2^{10}}\right)$

77. If a, b, c are in A.P. and if their squares taken in the same order form a G.P., then $(a + c)^4 =$
(a) $16a^2c^2$ (b) $4a^2c^2$
(c) $8a^2c^2$ (d) $2a^2c^2$
(e) a^2c^2

78. The value of x satisfying the relation $11({}^xC_3) = 24({}^{(x+1)}C_2)$ is
(a) 8 (b) 9 (c) 11 (d) 10
(e) 12

79. The power of x in the term with the greatest coefficient in the expansion of $\left(1+\frac{x}{2}\right)^{10}$ is
(a) 2 (b) 3 (c) 4 (d) 5
(e) 6

80. The number of 5-digit numbers (no digit is repeated) that can be formed by using the digits 0, 1, 2, ..., 7 is
(a) 1340 (b) 1860 (c) 2340 (d) 2160
(e) 3200

81. If m_1 and m_2 satisfy the relation ${}^{m+5}P_{m+1} = \frac{11}{2}(m-1)({}^{m+3}P_m)$, then $m_1 + m_2$ is equal to
(a) 10 (b) 9 (c) 13 (d) 17
(e) 15

82. Sum of coefficients of the last 6 terms in the expansion of $(1 + x)^{11}$ when the expansion is in ascending powers of x, is
(a) 2048 (b) 32 (c) 512 (d) 64
(e) 1024

83. If $C_0, C_1, C_2, ..., C_{15}$ are binomial coefficients in $(1 + x)^{15}$, then $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + ... + 15\frac{C_{15}}{C_{14}} =$
(a) 60 (b) 120 (c) 64 (d) 124
(e) 144

84. If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 6 & 12 \end{vmatrix}$ and $\Delta' = \begin{vmatrix} 4 & 8 & 15 \\ 3 & 6 & 12 \\ 2 & 3 & 5 \end{vmatrix}$, then
(a) $\Delta' = 2\Delta$ (b) $\Delta' = -2\Delta$
(c) $\Delta' = \Delta$ (d) $\Delta' = -\Delta$
(e) $\Delta' = 3\Delta$

85. The roots of the equation $\begin{vmatrix} 1+x & 3 & 5 \\ 2 & 2+x & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$ are

(a) 2, 1, –9 (b) 1, 1, –9
(c) –1, 1, –9 (d) –2, –1, –8
(e) –2, 1, 1

86. If $\begin{pmatrix} 0 & 3 & 2b \\ 2 & 0 & 1 \\ 4 & -1 & 6 \end{pmatrix}$ is singular, then the value of b is equal to

(a) –3 (b) 3 (c) –6 (d) 6
(e) –2

87. If $A = \begin{bmatrix} x & x-1 \\ 2x & 1 \end{bmatrix}$ and if det A = –9, then the values of x are

(a) $\frac{3}{2}, -3$ (b) $\frac{-2}{3}, 3$
(c) $\frac{2}{3}, 3$ (d) $\frac{-3}{2}, 3$
(e) $\frac{-3}{2}, -3$

88. The value of the determinant $\begin{vmatrix} \cos^2 54^\circ & \cos^2 36^\circ & \cot 135^\circ \\ \sin^2 53^\circ & \cot 135^\circ & \sin^2 37^\circ \\ \cot 135^\circ & \cos^2 25^\circ & \cos^2 65^\circ \end{vmatrix}$ is equal to

(a) –2 (b) –1 (c) 0 (d) 1
(e) 2

89. If A and B are square matrices of the same order and if $A = A^T$, $B = B^T$, then $(ABA)^T$ =

(a) BAB (b) ABA
(c) $ABAB$ (d) AB^T
(e) $(AB)^T$

90. If $|x - 3| < 2x + 9$, then x lies in the interval

(a) $(-\infty, -2)$ (b) $(-2, 0)$
(c) $(-2, \infty)$ (d) $(2, \infty)$
(e) $(-12, -2)$

91. The area and perimeter of a rectangle are A and P respectively. Then P and A satisfy the inequality

(a) $P + A > PA$ (b) $P^2 \le A$
(c) $A - P < 2$ (d) $P^2 \le 4A$
(e) $P^2 \ge 16A$

92. For any two statements p and q, the statement $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to

(a) $\sim p$ (b) p
(c) q (d) $\sim q$
(e) $p \vee q$

93. Let p, q, r be three statements. Then $\sim (p \vee (q \wedge r))$ is equal to

(a) $(\sim p \wedge \sim q) \wedge (\sim p \wedge \sim r)$
(b) $(\sim p \vee \sim q) \wedge (\sim p \vee \sim r)$
(c) $(\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$
(d) $(\sim p \vee \sim q) \vee (\sim p \wedge \sim r)$
(e) $(\sim p \wedge \sim q) \vee (\sim p \vee \sim r)$

94. Consider the two statements P : He is intelligent and Q : He is strong. Then the symbolic form of the statement "It is not true that he is either intelligent or strong" is

(a) $\sim P \vee Q$ (b) $\sim P \vee \sim Q$
(c) $\sim P \wedge Q$ (d) $P \vee \sim Q$
(e) $\sim (P \vee Q)$

95. If $\tan\frac{\theta}{2} = \frac{1}{2}$, then the value of $\sin\theta$ is

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) 1
(e) $\frac{2}{5}$

96. If $x = 5 + 2\sec\theta$ and $y = 5 + 2\tan\theta$, then $(x - 5)^2 - (y - 5)^2$ is equal to

(a) 3 (b) 1 (c) 0 (d) 4
(e) 2

97. The value of tan 15° + tan 75° is equal to

(a) $2\sqrt{3}$ (b) 2
(c) $2 - \sqrt{3}$ (d) $4\sqrt{3}$
(e) 4

98. The period of the function $f(x) = \cos 4x + \tan 3x$ is

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) π
(e) 2π

99. If $\sin(\theta + \phi) = n \sin(\theta - \phi)$, $n \ne 1$, then the value of $\frac{\tan\theta}{\tan\phi}$ is equal to

(a) $\frac{n}{n-1}$ (b) $\frac{n+1}{n-1}$ (c) $\frac{n}{1-n}$ (d) $\frac{n-1}{n+1}$
(e) $\frac{1+n}{1-n}$

100. $2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{4}\right)=$

(a) $\tan^{-1}\left(\frac{16}{13}\right)$ (b) $\tan^{-1}\left(\frac{17}{23}\right)$

(c) $\frac{\pi}{4}$ (d) 0

(e) $\tan^{-1}\left(\frac{11}{12}\right)$

101. If $\cos^{-1}x+\cos^{-1}y=\frac{2\pi}{7}$, then the value of $\sin^{-1}x+\sin^{-1}y$ is equal to

(a) $\frac{4\pi}{7}$ (b) $\frac{3\pi}{7}$ (c) $\frac{2\pi}{7}$ (d) $\frac{6\pi}{7}$

(e) $\frac{5\pi}{7}$

102. If $\cos^{-1}x > \sin^{-1}x$, then x lies in the interval

(a) $\left(\frac{1}{2},1\right]$ (b) $(0, 1]$

(c) $\left[-1,\frac{1}{\sqrt{2}}\right)$ (d) $[-1, 1]$

(e) $[0, 1]$

103. If $0 \le x \le 2\pi$, then the number of solutions of the equation $\sin^8x + \cos^6x = 1$ is

(a) 2 (b) 3 (c) 4 (d) 5

(e) 8

104. Let $A(0, 0)$ and $B(8, 0)$ be two vertices of a right angled triangle whose hypotenuse is BC. If the circumcentre is $(4, 2)$, then the point C is

(a) $(2, 4)$ (b) $(0, 8)$

(c) $(0, 4)$ (d) $(0, 6)$

(e) $(2, 8)$

105. If coordinates of the circumcentre and the orthocentre of a triangle are respectively $(5, 5)$ and $(2, 2)$, then the coordinates of the centroid are

(a) $(1, 1)$ (b) $(3, 1)$ (c) $(3, 3)$ (d) $(2, 2)$

(e) $(4, 4)$

106. If the points $A(3, 4)$, $B(x_1, y_1)$ and $C(x_2, y_2)$ are such that both $3, x_1, x_2$ and $4, y_1, y_2$ are in A.P., then

(a) A, B, C are vertices of an isosceles triangle

(b) A, B, C are collinear points

(c) A, B, C are vertices of a right angled triangle

(d) A, B, C are vertices of a scalene triangle

(e) A, B, C are vertices of an equilateral triangle

107. If p_1 and p_2 are respectively lengths of perpendicular from the origin to the straight lines $x\sec\theta + y\,\mathrm{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos2\theta$, then $4p_1^2 + p_2^2 =$

(a) 1 (b) a^2 (c) $\frac{1}{a^2}$ (d) a

(e) $\frac{1}{a}$

108. The distance between the point $(1, 2)$ and the point of intersection of the lines $2x + y = 2$ and $x + 2y = 2$ is

(a) $\frac{\sqrt{17}}{3}$ (b) $\frac{\sqrt{16}}{3}$ (c) $\frac{\sqrt{17}}{5}$ (d) $\frac{\sqrt{19}}{3}$

(e) $\frac{\sqrt{19}}{5}$

109. If a straight line is perpendicular to $2x + 8y = 10$ and meets the x-axis at $(5, 0)$, then it meets the y-axis at

(a) $(0, -2)$ (b) $(0, -8)$

(c) $(0, -10)$ (d) $(0, -16)$

(e) $(0, -20)$

110. If the straight line $5x + y = k$ forms a triangle with the coordinate axes of area 10 sq. units, then the values of k are

(a) ±15 (b) ±10 (c) ±5 (d) ±20

(e) ±7

111. A circle passes through the point $(6, 2)$. If segments of the straight lines $x + y = 6$ and $x + 2y = 4$ are two diameters of the circle, then its radius is

(a) 4 (b) 8 (c) $\sqrt{5}$ (d) $2\sqrt{5}$

(e) $4\sqrt{5}$

112. The parametric form of equation of the circle $x^2 + y^2 - 6x + 2y - 28 = 0$ is

(a) $x=-3+\sqrt{38}\cos\theta,\ y=-1+\sqrt{38}\sin\theta$

(b) $x=\sqrt{28}\cos\theta,\ y=\sqrt{28}\sin\theta$

(c) $x=-3-\sqrt{38}\cos\theta,\ y=-1+\sqrt{38}\sin\theta$

(d) $x=3+\sqrt{38}\cos\theta,\ y=-1+\sqrt{38}\sin\theta$

(e) $x = 3 + 38\cos\theta,\ y = -1 + 38\sin\theta$

113. The point on the circle $(x - 1)^2 + (y - 1)^2 = 1$ which is nearest to the circle $(x - 5)^2 + (y - 5)^2 = 4$ is

(a) $(2, 2)$ (b) $\left(\frac{3}{2},\frac{3}{2}\right)$

(c) $\left(\frac{\sqrt{2}+1}{\sqrt{2}},\frac{\sqrt{2}+1}{\sqrt{2}}\right)$ (d) $(\sqrt{2},\sqrt{2})$

(e) $(1+\sqrt{2},1+\sqrt{2})$

114. The line segment joining (5, 0) and (10 cosθ, 10 sinθ) is divided internally in the ratio 2 : 3 at P. If θ varies, then the locus of P is

(a) a pair of straight lines
(b) a circle
(c) a straight line
(d) a parabola
(e) an ellipse

115. $ABCD$ is a square with side a. If AB and AD are along the coordinate axes, then the equation of the circle passing through the vertices A, B and D is

(a) $x^2 + y^2 = \sqrt{2}a(x + y)$

(b) $x^2 + y^2 = \frac{a}{\sqrt{2}}(x + y)$

(c) $x^2 + y^2 = a(x + y)$
(d) $x^2 + y^2 = a^2(x + y)$
(e) $a(x^2 + y^2) = x + y$

116. The distance between the directrices of the hyperbola $x^2 - y^2 = 9$ is

(a) $\frac{9}{\sqrt{2}}$ (b) $\frac{5}{\sqrt{3}}$ (c) $\frac{3}{\sqrt{2}}$ (d) $3\sqrt{2}$
(e) $5\sqrt{3}$

117. If the line $y = kx$ touches the parabola $y = (x - 1)^2$, then the values of k are

(a) 2, −2 (b) 0, 4
(c) 0, −2 (d) 0, 2
(e) 0, −4

118. If the semi-major axis of an ellipse is 3 and the latus rectum is $\frac{16}{9}$, then the standard equation of the ellipse is

(a) $\frac{x^2}{9} + \frac{y^2}{8} = 1$ (b) $\frac{x^2}{8} + \frac{y^2}{9} = 1$
(c) $\frac{x^2}{9} + \frac{3y^2}{8} = 1$ (d) $\frac{3x^2}{8} + \frac{y^2}{9} = 1$
(e) $\frac{x^2}{9} + \frac{8y^2}{3} = 1$

119. If a point $P(x, y)$ moves along the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and if C is the centre of the ellipse, then, $4\max\{CP\} + 5\min\{CP\} =$

(a) 25 (b) 40 (c) 45 (d) 54
(e) 16

120. The one end of the latus rectum of the parabola $y^2 - 4x - 2y - 3 = 0$ is at

(a) (0, −1) (b) (0, 1)
(c) (0, −3) (d) (3, 0)
(e) (0, 2)

SOLUTIONS

1. (e) : Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}$

If θ is angle between two vectors, then

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{2 - 2 + 2}{\sqrt{3}\sqrt{12}} = \frac{2}{\sqrt{3}\cdot 2\sqrt{3}} = \frac{2}{6} = \frac{1}{3}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

2. (c) : Since given vectors are coplanar

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0 \Rightarrow \beta = 1$$

$$\Rightarrow \quad \text{Since } |\vec{c}| = \sqrt{3}$$

$$\Rightarrow \quad \sqrt{1 + \alpha^2 + \beta^2} = \sqrt{3} \Rightarrow \alpha^2 + \beta^2 = 2 \Rightarrow \alpha^2 = 1$$

$$\therefore \quad \alpha = \pm 1$$

3. (b) : $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} \Rightarrow \overrightarrow{PQ} + \overrightarrow{OP} = \overrightarrow{OQ}$

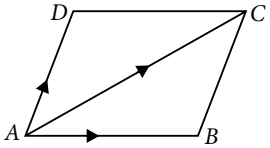
$$\therefore \quad \left|\overrightarrow{PQ} + \overrightarrow{OP}\right| = \left|\overrightarrow{OQ}\right| = \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{14}$$

4. (c) : $\overrightarrow{AD} = \overrightarrow{BC} = 2\hat{i} + 3\hat{j} - 5\hat{k}$

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 3\hat{i} + 6\hat{j} + 2\hat{k}$$

$$\therefore \quad \vec{p} = \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|}$$

$$= \frac{1}{7}(3\hat{i} + 6\hat{j} + 2\hat{k})$$



5. (d) : Perpendicular distance from $\vec{a}_2$ on $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ is

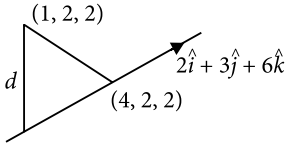
$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

where $\vec{a}_1 = 4\hat{i} + 2\hat{j} + 2\hat{k}$

$$\vec{a}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$|\vec{b}| = \sqrt{4 + 9 + 36} = 7$$



$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ 2 & 3 & 6 \end{vmatrix} = -18\hat{j} + 9\hat{k}$$

$$|(\vec{a}_2 - \vec{a}_1) \times \vec{b}| = \sqrt{(18)^2 + (9)^2} = \sqrt{405}$$

6. (c)

7. (e) : $\vec{Q} = \dfrac{\vec{a} + \vec{b} + \vec{c}}{3}$

$\therefore \quad \overrightarrow{QA} = \vec{a} - \dfrac{\vec{a} + \vec{b} + \vec{c}}{3} = \dfrac{2\vec{a} - \vec{b} - \vec{c}}{3}$

$\therefore \quad \overrightarrow{QA} + \overrightarrow{QB} + \overrightarrow{QC} = \vec{0}$

8. (d) : Given lines are

$\dfrac{x}{\frac{1}{2}} = \dfrac{y}{\frac{1}{3}} = \dfrac{z}{-1}$ and $\dfrac{x}{\frac{1}{6}} = \dfrac{y}{-1} = \dfrac{z}{\frac{-1}{4}}$

Let $< a_1, b_1, c_1 > = < \dfrac{1}{2}, \dfrac{1}{3}, -1 >$

and $< a_2, b_2, c_2 > = < \dfrac{1}{6}, -1, \dfrac{-1}{4} >$

are direction ratios of given lines.

since $\left(\dfrac{1}{2} \cdot \dfrac{1}{6}\right) + \left(\dfrac{1}{3} \times -1\right) + \left(-1 \times \dfrac{-1}{4}\right) = 0 \Rightarrow \quad \theta = \dfrac{\pi}{2}$

9. (e) : Projection of $\vec{a}$ on $\vec{b}$

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{(3\hat{i} - 1\hat{j} + 5\hat{k}) \cdot (2\hat{i} + 4\hat{j} + 4\hat{k})}{\sqrt{(2)^2 + (4)^2 + (4)^2}} = \frac{22}{6} = \frac{11}{3}$$

10. (a) : If θ is the required angle, then

$\sin\theta$ = cosine of angle between $\hat{i} - \hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + \hat{k}$

$$= \frac{2 + 1 + 1}{\sqrt{3}\sqrt{6}} = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

11. (c) : $\dfrac{1 - \tan^2\alpha}{1 + \tan^2\alpha} + \dfrac{1}{\sec 2\beta} - 2\sin^2 y =$

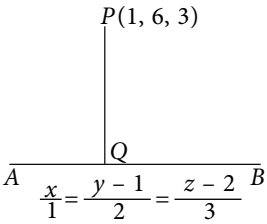
$$= \cos 2\alpha + \cos 2\beta + 1 - 2\sin^2 y - 1$$

Since $\cos 2\alpha + \cos 2\beta + \cos 2y = -1$

$\therefore$ Required value $= -2$

12. (d) : Midpoint of line joining (3, 2, 6) and (5, 4, 8) is (4, 3, 7) satisfying $x + y + z = 14$

13. (a) : Let Q be the foot of $\perp$r PQ



lie on line $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$

Let $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3} = k$(say)

$\Rightarrow \quad x = k,\ y = 2k + 1,\ z = 3k + 2$

$\Rightarrow$ Point $Q = (k, 2k + 1, 3k + 2)$ lie on line AB.

Since $PQ \perp AB$

$\therefore \quad (k - 1)\cdot 1 + (2k + 1 - 6)\cdot 2 + (3k + 2 - 3)\cdot 3 = 0$

$\Rightarrow \quad k - 1 + 2(2k - 5) + (3k - 1)\cdot 3 = 0$

$\Rightarrow \quad k - 1 + 4k - 10 + 9k - 3 = 0$

$\Rightarrow \quad 14k - 14 = 0 \Rightarrow k = 1$

$\therefore$ Required point $Q = (1, 3, 5)$

14. (d)

15. (a) : $\cos^2 60° + \cos^2 45° + \cos^2\alpha = 1$

$$\Rightarrow \quad \frac{1}{4} + \frac{1}{2} + \cos^2\alpha = 1$$

$$\Rightarrow \quad 1 - \cos^2\alpha = \frac{3}{4} \quad \Rightarrow \quad \sin^2\alpha = \frac{3}{4}$$

16. (e) : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A) + P(B) = P(A \cup B) + P(A \cap B)$

$\therefore \quad 1 + P(A \cap B) - P(B) - P(A)$

$= 1 + P(A \cap B) - P(A \cup B) - P(A \cap B)$

$= 1 - P(A \cup B) = P(A \cup B)^c = P(\bar{A} \cap \bar{B})$

17. (e) : Let the unknown data be a and b

$$\text{Variance} = \frac{1}{n}\sum x^2 - \bar{x}^2 = 5.2$$

$$\Rightarrow \quad \frac{1}{5}(4 + 16 + 36 + a^2 + b^2) - 16 = 5.2$$

$\Rightarrow \quad a^2 + b^2 = 50$ ...(i)

Also $2 + 4 + 6 + a + b = 20 \Rightarrow a + b = 8$ ...(ii)

It is clear that the two observations are 1 and 7 which statisfies both (i) and (ii).

18. (b) : $a = 148$; $d = -2$

sum of first n terms $= \dfrac{n}{2}(296 + (n-1)(-2))$

Since A. M. $= \dfrac{125}{n}$

$\Rightarrow$ sum of n terms $= n \times 125$

$\therefore \quad 298 - 2n = 250 \Rightarrow n = 24$

19. (c) : There are a total of 18 observations

$\therefore$ combined mean $= \dfrac{(a_1 \times 10) + (a_2 \times 8)}{18}$

where a_1 and a_2 denote the means of two groups respectively.

But $a_1 = 15 \Rightarrow \quad \dfrac{40}{3} = \dfrac{150 + 8a_2}{18} \quad \Rightarrow \quad a_2 = \dfrac{45}{4}$

20. (b) : $f(256) = f(16^2) = f(16) + 6$

$f(16) = f(4^2) = f(4) + 6,\ f(4) = f(2^2) = f(2) + 6$

But $f(2) = 8$

$\therefore\ f(4) = 14\ ; f(16) = 20$ and $f(256) = 26$

21. (d) : $\frac{10}{9}\cdot\frac{9}{10}\cdot\frac{8}{7}\cdot\frac{7}{8}\cdot\frac{6}{5}\cdot\frac{5}{6}\cdot\frac{4}{3}\cdot\frac{3}{4}\cdot\frac{1}{2}=\frac{1}{2}$

$\left(\text{Using } \lim_{x\to 0}\frac{\sin x}{x}=1\right)$

22. (b) : The domain is not explicitly given; it is $R' - \{0, \pm 1\}$ and the function is continuous on its domain.

23. (a) : Required limit $= \lim_{y\to\infty}\left[\frac{\sin\left(\frac{1}{y}\right)}{\left(\frac{1}{y}\right)}-\frac{1}{y}\right]=1-0=1$

24. (e) : $\lim_{x\to\infty}\frac{2x}{3(3x-2)}=\lim_{x\to\infty}\frac{2}{3\left(3-\frac{2}{x}\right)}=\frac{2}{9}$

25. (a) : Given, $f'(x) = g(x + 1)$

$\Rightarrow\ f''(x) = g'(x + 1)$. Also, $g'(x) = h(x - 1)$

$\Rightarrow\ g'(x + 1) = h(x) \Rightarrow f''(x) = h(x)$

$\Rightarrow\ f''(2x) = h(2x)$

26. (e) : $f(x)=\sin^{-1}\left(\frac{1-\cos 2x}{2\sin x}\right)$

$= \sin^{-1}(\sin x)$ (using $\cos 2x = 1 - 2\sin^2 x$)

$\Rightarrow\ f'(x) = \pm 1 \Rightarrow |f'(x)| = 1$

27. (d) : $y^2 = 100 + 45 = 145$

$(\because\ \tan^{-1}x + \cot^{-1}x = 1,\ \sec^{-1}x + \operatorname{cosec}^{-1}x = 1)$

$\Rightarrow\ 2yy' = 0 \Rightarrow y' = 0 \quad (\because\ y \neq 0)$

28. (b) : The required limit $= \lim_{x\to 0}(3x-7) = -7$

29. (a) : $y = \sec(\tan^{-1}x)$

$=\sec\left(\sec^{-1}\sqrt{1+x^2}\right)$

$\Rightarrow\ y=\sqrt{1+x^2} \Rightarrow \frac{dy}{dx}=\frac{x}{\sqrt{1+x^2}}$

30. (e) : If $t = \tan\theta$, then

$\sin x=\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta,\ \tan y=\frac{2\tan\theta}{1-\tan^2\theta}=\tan 2\theta$

$\therefore\ x = 2\theta,\ y = 2\theta$

$\Rightarrow\ y = x \Rightarrow \frac{dy}{dx}=1$

31. (a) : We have, $y=1-\frac{1}{x+1}+1+\frac{1}{x}$

$\Rightarrow\ \frac{dy}{dx}=\frac{1}{(1+x)^2}-\frac{1}{x^2}$

$\Rightarrow\ \frac{d^2y}{dx^2}=\frac{-2}{(1+x)^3}+\frac{2}{x^3}$

$=-\frac{1}{4}+2$ at $x=1=\frac{7}{4}$

32. (c) : $[3\sin^2(10x + 11) - 7]^2$ is maximum

When $\sin^2(10x + 11)$ is minimum

$\because$ Minimum value of $3\sin^2(10x + 11) = 0$

$\therefore$ Maximum value $= (-7)^2 = 49$.

33. (a) : If A sq. units is the area measure when the radius is r units, then $A = \pi r^2$

$\therefore\ \frac{dA}{dt}=2\pi r\frac{dr}{dt}$ But $\frac{dA}{dt}=3c\frac{dr}{dt}$

$\therefore\ 2\pi r = 3c \quad \therefore\ c = 4\pi$, when $r = 6$

34. (b) : $\frac{dy}{dx}=6x-5$

$= 6 - 5 = 1$ at $(1, 4)$

35. (d) : $6x-11=\frac{227-5}{11-5}=\frac{222}{6}=37$

$\Rightarrow\ 6x = 48 \Rightarrow x = 8$

36. (b) : $f'(x) = \cos x - k$

Hence, f decreases if $\cos x \le k$

$\Rightarrow$ for decreasing $k \ge 1 \qquad [\because\ -1 \le \cos x \le 1]$

37. (e) : We have, $y = m\log x + nx^2 + x$

$\Rightarrow\ \frac{dy}{dx}=\frac{m}{x}+2nx+1$

$\Rightarrow\ \frac{m}{2}+4n+1=0$ at $x=2$ …(i)

$m + 2n + 1 = 0$ at $x = 1$ …(ii)

from (i) and (ii), we get $6n + 1 = 0$

$\Rightarrow\ n=-\frac{1}{6}\ \&\ m=-\frac{2}{3}$

$\Rightarrow\ 2m+10n=\frac{-4}{3}-\frac{5}{3}=-3$

38. (b) : Acceleration $=\frac{d^2s}{dt^2} = 12t - 12$

acceleration is zero when $t = 1$

velocity at time $t = \frac{ds}{dt} = 6t^2 - 12t + a$

$\therefore\ -3 = 6 - 12 + a$

$\Rightarrow\ a = 3$

39. (e) : $\int \frac{(1+x)e^x dx}{\cot(x e^x)} = \int \frac{du}{\cot u}$ if $u = x e^x$

$= \log|\sec u| + C$

$= \log|\sec(x e^x)| + C$

40. (d) : $\int \frac{x^5 dx}{\sqrt{1+x^3}} = \frac{1}{3}\int \frac{(u-1)}{\sqrt{u}} du$ if $u = 1 + x^3$

$= \frac{1}{3}\int\left(\sqrt{u} - \frac{1}{\sqrt{u}}\right)du$

$= \frac{1}{3}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right] + C$

$= \frac{2\sqrt{1+x^3}}{9}(1 + x^3 - 3) + C$

$= \frac{2}{9}\sqrt{1+x^3}\,(x^3 - 2) + C$

41. (e) : $\int \frac{4e^x}{2e^x - 5e^{-x}} dx = \int \frac{4e^{2x}}{2e^{2x} - 5} dx$

$= \int \frac{du}{u}$ if $u = 2e^{2x} - 5$

$= \log|u| + C$

$= \log|2\,e^{2x} - 5| + C$

42. (a) : $\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx = \int\left(x + 2 + \frac{1}{x}\right)dx$

$= \frac{1}{2}x^2 + 2x + \log|x| + C$

43. (a) : $\int \frac{x^{n-1}}{x^{2n} + a^2} dx = \frac{1}{n}\int \frac{du}{a^2 + u^2}$; if $u = x^n$

$= \frac{1}{n}\cdot\frac{1}{a}\tan^{-1}\frac{u}{a} + C \;= \frac{1}{na}\tan^{-1}\left(\frac{x^n}{a}\right) + C$

44. (c) : $\int \frac{(x+1)^2}{x(x^2+1)} dx = \int\left(\frac{1}{x} + \frac{2}{x^2+1}\right)dx$

$= \log|x| + 2\tan^{-1}x + C$

45. (e) : $\int \frac{dx}{x \log x^2} = \int \frac{du}{2u}$ (if $u = \log x$)

$= \frac{1}{2}\log|u| + C' = \frac{1}{2}\log|\log x| + C'$

$= \frac{1}{4}\log|\log x^2| + C$

46. (b) : $\int_0^1 \frac{dx}{(x^2+16)(x^2+25)}$

$= \frac{1}{9}\int_0^1 \left(\frac{1}{(x^2+16)} - \frac{1}{(x^2+25)}\right)dx$

$= \frac{1}{9}\cdot\left(\frac{1}{4}\tan^{-1}\frac{x}{4} - \frac{1}{5}\tan^{-1}\frac{x}{5}\right)_0^1$

$= \frac{1}{9}\left[\frac{1}{4}\tan^{-1}\frac{1}{4} - \frac{1}{5}\tan^{-1}\frac{1}{5}\right]$

47. (e) : $\int_{-1}^{1} x(1-x)(1+x)dx = 0$

Using property $\int_{-a}^{a} f(x)dx = 0$, if $f(x)$ is an odd function.

Since $f(x) = x(1 - x)(1 + x)$ is an odd function

$[f(-x) = -x(1 + x)(1 - x) = -f(x)]$

48. (b) : Point of intersection of $y = x^2 + 3$ & $y = 2x + 3$ are $x = 0$ or 2

$\therefore$ Required area measure $= \left|\int_0^2 (x^2 - 2x)dx\right|$

$= \left|\frac{8}{3} - 4\right| = \frac{4}{3}$ sq. units

49. (c)

50. (b) : $\int_0^{\sqrt{\pi/2}} 2x^3 \sin x^2 dx = \int_0^{\pi/4} x \sin x\, dx$

(By Putting $x^2 = t$)

$= [x(-\cos x) - (-\sin x)]_0^{\pi/4}$

(Using Intergration by parts)

$= -\frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}\left(1 - \frac{\pi}{4}\right)$

51. (a) : $x^2 + y^2 = 1$

Differentiating w.r.t. 'x', we get

$2x + 2yy' = 0$

Again differentiating w.r.t. x, we get

$1 + (y')^2 + yy'' = 0$

52. (a) : We have, $y'(y^2 - x) = y$

$\Rightarrow \frac{dx}{dy} + \frac{1}{y}x = y$

I.F. $= e^{\int \frac{1}{y}dy} = y$

Required solution is $x \cdot y = \int y^2 dy + A$

or $3xy = y^3 - C$

53. (c) : On squaring the equation, we get

$$\left[\frac{2d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^3y}{dx^3}\right)^2$$

$\therefore$ Order = 3, degree = 2

54. (a) : We have, $\frac{dy}{dx} = y + \frac{y}{x}$

$$\Rightarrow \frac{dy}{y} = \frac{dx}{x} + dx$$

$\Rightarrow \log|y| = \log|x| + x + a$

$\Rightarrow y = e^{\log|x|}.e^x.e^a = Cx\,e^x$

55. (c) : For function to be defined,

$7 - 3x \geq 0 \Rightarrow x \leq \frac{7}{3}$ and $x > 0$

$$\Rightarrow \quad 0 < x \leq \frac{7}{3}$$

56. (b) : $f(2n + 1) = -1$ for all n

$\because$ $f(4) = f(8) = f(16) = 1$

$f(1) = f(2) = 1$ and $f(2n) = 1$

$f(2n) = -1$ when n is odd

$\therefore$ sum = 5 – 20 = –15.

57. (c)

58. (b)

59. (e) : Since, $-1 \leq \cos 3x \leq 1$

$\Rightarrow 1 \leq 2 - \cos 3x \leq 3$

$$\Rightarrow \frac{1}{3} \leq \frac{1}{2 - \cos 3x} \leq 1$$

60. (c) : $n(C) = 40$, $n(F) = 16$, $n(H) = 11$

$n(C \cap F) = 8$, $n(C \cap H) = 6$

$n(F \cap H) = 2$, $n(C \cap F \cap H) = 1$

$n(C \cup F \cup H) = 67 - 16 + 1$

$= 52$

$n(C' \cap F' \cap H') = 80 - 52 = 28$

61. (b) : $\text{Re}(1 + iy)^3 = 1 - 3y^2 = -26$

$\Rightarrow y^2 = 9 \Rightarrow |y| = 3.$

62. (e) : $\sqrt{x^2 + y^2} = -y + 1$

$\Rightarrow x^2 + y^2 = y^2 - 2y + 1$

$\Rightarrow x^2 = 1 - 2y.$

63. (a) : $i^{10} + i^{11} + i^{12} + i^{13} = 0$

and $\left(\frac{1}{i^{11}} + \frac{1}{i^{12}} + \frac{1}{i^{13}} + \frac{1}{i^{14}}\right) = 0$

$\therefore$ Remaining expression $= i^{14} + \frac{1}{i^{15}} = i^2 + \frac{1}{i^3} = -1 + i$

64. (d) : $f(z) = 1 + z + z^2$

$\text{Re}(f(z)) = 1 + x^2 - y^2 + x$

$\text{Re}(\overline{f(z)}) = 0 \Rightarrow x^2 - y^2 + x + 1 = 0$

65. (a) : $z^2 = 2i$

$z^2\, e^{z-i} = 2i\, e^{1+i-i}$

$= 2i\, e$

$= 2e[i]$

$\therefore$ Arg of $z^2 e^{z-i} = \frac{\pi}{2}$

66. (d) : $\sqrt{4p^2 - 4q} = 2\sqrt{q^2 - p}$

$\Rightarrow 4p^2 - 4q = 4(q^2 - p)$

$\Rightarrow p^2 - q^2 + p - q = 0$

$\Rightarrow p + q + 1 = 0$

67. (c) : We have, $|x - 3|^2 + |x - 3| - 2 = 0$

$\Rightarrow |x - 3| = 1$

$\Rightarrow x - 3 = \pm 1 \Rightarrow x = 2, 4$

68. (a) : $\alpha + \beta = -\frac{3}{2} \Rightarrow 3(\alpha + \beta) = -\frac{9}{2}$

$\alpha\beta = \frac{5}{2} \Rightarrow 9\alpha\beta = \frac{45}{2}$

$\therefore$ Required equation is, $x^2 + \frac{9}{2}x + \frac{45}{2} = 0$

or $2x^2 + 9x + 45 = 0.$

69. (e) : Let $y = \frac{x}{x^2 - 5x + 9}$

or $x^2y - (5y + 1)x + 9y = 0$

For real x, $\Delta \geq 0$

i.e. $(5y + 1)^2 - 36y^2 \geq 0$

$\Rightarrow (5y + 1 - 6y)(5y + 1 + 6y) \geq 0$

$\Rightarrow (-y + 1)(11y + 1) \geq 0$

$\Rightarrow (y - 1)(11y + 1) \leq 0$

$$\Rightarrow \quad y \in \left[\frac{-1}{11}, 1\right]$$

70. (d) : $3x^2 - 2(a + b + c)x + ab + bc + ca = 0$

$\Rightarrow 4(a + b + c)^2 = 12(ab + bc + ca)$ $\quad (\because D = 0)$

$\Rightarrow (a + b + c)^2 = 3(ab + bc + ca)$

$\Rightarrow a^2 + b^2 + c^2 = ab + bc + ca.$

71. (e) : One root $= \frac{1}{1 + \sqrt{3}} = \frac{1 - \sqrt{3}}{-2} = -\frac{1}{2} + \frac{\sqrt{3}}{2}$

$\therefore$ Other root $= \frac{-1}{2} - \frac{\sqrt{3}}{2}$

Sum = –1, product $= \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$

$\therefore$ Required equation is $x^2 + x - \frac{1}{2} = 0$

or $2x^2 + 2x - 1 = 0.$

72. (a) : $a_5 = ar^4 = 1458,\ a_8 = ar^7 = 54$

$\Rightarrow \quad r^3 = \frac{1}{27} \Rightarrow r = \frac{1}{3}$

73. (d) **74. (d)**

75. (d) : $a_4 = ar^3 = 32,\ r = \frac{a}{2}$

$\Rightarrow a^4 = 256 \Rightarrow a = 4$

$\Rightarrow r = 2$

$\Rightarrow a_{15} = 4 \times 2^{14} = 2^{16}.$

76. (b) : $25 + \frac{50}{4} + \frac{25}{4} + ... +$ area of S_{10} (G.P series)

where $a = 25,\ r = \frac{1}{2}$

$$\Rightarrow \quad \text{Sum of G.P series} = \frac{25\left[1 - \frac{1}{2^{10}}\right]}{1 - \frac{1}{2}} = 50\left[1 - \frac{1}{2^{10}}\right]$$

77. (a) : $(a + c)^4 = (2b)^4 = 16a^2c^2 (\because b^2 = ac)$

78. (d) : $\frac{11}{3} = \frac{24(x+1)}{(x-2)(x-1)}$

Only $x = 10$ satisfy it.

79. (b) : $\left(1 + \frac{x}{2}\right)^{10} = 1 + {}^{10}C_1 . \frac{1}{2} . x + {}^{10}C_2 . \frac{1}{4} . x^2 + {}^{10}C_3 . \frac{1}{8} . x^3 + {}^{10}C_4 . \frac{1}{16} . x^4 + ...$

By inspection, ${}^{10}C_3 . \frac{1}{8}$ is the highest coefficient.

$\therefore$ Power of x is 3.

80. (*) : No. of ways $= 7 \times 7 \times 6 \times 5 \times 4 = 5880$

81. (c) : $(m + 4)\ (m + 5) = 22(m - 1)$ (using given relation)

$\Rightarrow \quad m^2 - 13m + 42 = 0$

$\Rightarrow \quad m = 7, 6 \Rightarrow m_1 + m_2 = 13$

82. (e) : Sum of coeff. of last 6 terms

$= \frac{2^{11}}{2} = 2^{10} = 1024$

(Using relation ${}^nC_0 + {}^nC_1 + ... + {}^nC_n = 2^n$)

83. (b) : Since $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n - (r-1)}{r}$

$\Rightarrow \quad r\frac{{}^nC_r}{{}^nC_{r-1}} = n + 1 - r$

$\Rightarrow \quad \sum_{r=1}^{15} r\frac{{}^nC_r}{{}^nC_{r-1}} = \sum_{r=1}^{15}(16 - r)$

$= 16 \times 15 - \frac{1}{2} \times 15 \times 16 = 240 - 120 = 120$

84. (d) : $\Delta = (36 - 30) - 2(24 - 15) + 3(12 - 9) = -3$

$\Delta' = 4(-6) - 8(-9) + 15(-3)$

$= 3 = -\Delta.$

85. (b) : We have, $\begin{vmatrix} 1+x & 3 & 5 \\ 2 & 2+x & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$

Apply $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} 9+x & 3 & 5 \\ 9+x & 2+x & 5 \\ 9+x & 3 & x+4 \end{vmatrix} = 0$$

Apply $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$\Rightarrow \quad (9 + x)\begin{vmatrix} 1 & 3 & 5 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix} = 0$$

$\Rightarrow (9 + x)\ (x - 1)^2 = 0 \Rightarrow x = -9$ or $x = 1, 1$

86. (c) : Let $A = \begin{pmatrix} 0 & 3 & 2b \\ 2 & 0 & 1 \\ 4 & -1 & 6 \end{pmatrix}$

Since $|A| = 0$

$\Rightarrow -3(12 - 4) + 2b(-2) = 0$

$\Rightarrow -24 - 4b = 0 \Rightarrow 4b = -24$

$\Rightarrow b = -6$

87. (d) : $|A| = x - 2x(x - 1) = -9$

$\Rightarrow x - 2x^2 + 2x = -9$

$\Rightarrow 2x^2 - 3x - 9 = 0$

$$x = \frac{3 \pm \sqrt{9 + 72}}{4} = \frac{3 \pm 9}{4}$$

$$x = 3 \text{ or } \frac{-3}{2}$$

88. (c) : Apply $C_1 \to C_1 + C_2 + C_3$ and using $\cos(90 - \theta) = \sin\theta$, $\cos^2\theta + \sin^2\theta = 1$, we get

$$\begin{vmatrix} 0 & \cos^2 36° & \cot 135° \\ 0 & \cot 135° & \sin^2 37° \\ 0 & \cos^2 25° & \cos^2 65° \end{vmatrix} = 0$$

89. (b) : $(ABA)^T = A^TB^TA^T = ABA$ (Using $A = A^T,\ B = B^T$)

90. (c) : $|x - 3| < 2x + 9$

$\Rightarrow -(2x + 9) < (x - 3) < 2x + 9$

$\Rightarrow$ either $-2x - 9 < x - 3 \Rightarrow -3x < 6 \Rightarrow x > -2$ ...(i)

or $x - 3 < 2x + 9 \Rightarrow -x < 12 \Rightarrow x > -12$ ...(ii)

From (i) and (ii), we get $x > -2$

91. (e) : $A = xy$, $P = 2(x + y)$

Since $(x + y)^2 \geq 4xy$ (Using A.M. $\geq$ G.M. for real numbers x, y)

$$\Rightarrow \left(\frac{P}{2}\right)^2 \geq 4A \Rightarrow P^2 \geq 16A.$$

92. (a) : $\sim (p \vee q) \vee (\sim p \wedge q)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge q) = \sim p \wedge (\sim q \vee q) = \sim p$

93. (c) : $\sim[p \vee (q \wedge r)] = \sim p \wedge \sim(q \wedge r)$

$= \sim p \wedge (\sim q \vee \sim r)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$

94. (e)

95. (a) : $$\sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{2\times\frac{1}{2}}{1+\frac{1}{4}} = \frac{1}{\left(\frac{5}{4}\right)} = \frac{4}{5}$$

96. (d) : $(x - 5)^2 - (y - 5)^2 = 4\sec^2\theta - 4\tan^2\theta = 4$

97. (e) : $\tan 15° + \tan 75°$

$= 2 - \sqrt{3} + 2 + \sqrt{3} = 4$

98. (d) : $f(x) = \cos 4x + \tan 3x$

Period of $\cos 4x = \frac{2\pi}{4} = \frac{\pi}{2}$

Period of $\tan 3x = \frac{\pi}{3}$

$\therefore$ period of $f(x)$ = LCM of $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

$= \pi$

99. (b) : $\sin(\theta + \phi) = n \sin(\theta - \phi)$

$$\Rightarrow \frac{\sin(\theta+\phi)}{\sin(\theta-\phi)} = \frac{n}{1}$$

$$\Rightarrow \frac{Nr + Dr}{Nr - Dr} \Rightarrow \frac{\sin(\theta+\phi)+\sin(\theta-\phi)}{\sin(\theta+\phi)-\sin(\theta-\phi)} = \frac{n+1}{n-1}$$

$$\Rightarrow \frac{2\sin\theta\cos\phi}{2\cos\theta\sin\phi} = \frac{n+1}{n-1}$$

$$\Rightarrow \frac{\tan\theta}{\tan\phi} = \frac{n+1}{n-1}$$

100. (a) : $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4}$

$$= \tan^{-1}\left(\frac{2.\frac{1}{3}}{1-\frac{1}{9}}\right) + \tan^{-1}\left(\frac{1}{4}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{4}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{4}}{1-\frac{3}{4}.\frac{1}{4}}\right) = \tan^{-1}\left(\frac{16}{13}\right)$$

101. (e) : $\cos^{-1}x + \cos^{-1}y = \frac{2\pi}{7}$

$$\Rightarrow \left(\frac{\pi}{2} - \sin^{-1}x\right) + \left(\frac{\pi}{2} - \sin^{-1}y\right) = \frac{2\pi}{7}$$

$$\Rightarrow \sin^{-1}x + \sin^{-1}y = \pi - \frac{2\pi}{7} = \frac{5\pi}{7}$$

102. (c) : $\cos^{-1}x > \sin^{-1}x$

$$\Rightarrow \cos^{-1}x > \frac{\pi}{2} - \cos^{-1}x$$

$$\Rightarrow 2\cos^{-1}x > \frac{\pi}{2} \Rightarrow \cos^{-1}x > \frac{\pi}{4}$$

$$\Rightarrow x < \frac{1}{\sqrt{2}} \text{ and } x \geq -1$$

103. (d) : Given equation is $\sin^8 x + \cos^6 x = 1$

$\Rightarrow \sin^8 x + (1 - \sin^2 x)^3 = 1$

$\Rightarrow \sin^8 x - \sin^6 x + 3\sin^4 x - 3\sin^2 x = 0$

$\Rightarrow (\sin^4 x + 3)(\sin^4 x - \sin^2 x) = 0$

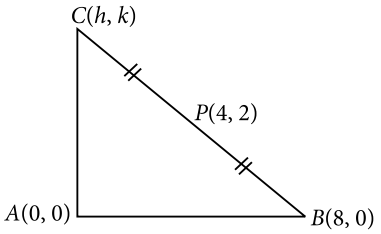
$\Rightarrow \sin^4 x = \sin^2 x$

$\Rightarrow$ either $\sin x = 0 \Rightarrow x = 0, \pi, 2\pi$ (3 solutions)

or $\sin^2 x = 1 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ (2 solutions)

$\therefore$ 5 solutions exist.

104. (c) : Let P be the circumcentre of the triangle



$$\Rightarrow \left(\frac{h+8}{2}, \frac{k+0}{2}\right) = (4, 2)$$

[In a right angled triangle, circumcentre is mid-point of hypotenuse]

$\Rightarrow$ $h = 0, k = 4$

$\therefore$ The point C is $(0, 4)$.

105. (e) : $O(2,2)$ —— G —— $C(5,5)$ (2 : 1)

$$G\left(\frac{10+2}{3}, \frac{10+2}{3}\right) = (4, 4)$$

(Using section formula)

106. (b) : We have, $x_1 - 3 = x_2 - x_1$ and $y_1 - 4 = y_2 - y_1$

[As both 3, x_1, x_2 and 4, y_1, y_2 are in A.P.]

$$\Rightarrow \quad \frac{y_1 - 4}{x_1 - 3} = \frac{y_2 - y_1}{x_2 - x_1}$$

$\therefore$ slope of AB = slope of BC

$\therefore$ A, B, C are collinear.

107. (b) : $$p_1 = \left|\frac{-a}{\sqrt{\sec^2\theta + \text{cosec}^2\theta}}\right| = a\sin\theta\cos\theta$$

$$= \frac{a}{2}\sin 2\theta$$

$$p_2 = \left|\frac{-a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}}\right| = a\cos 2\theta$$

$\Rightarrow$ $(2p_1)^2 + (p_2)^2 = a^2 \Rightarrow 4{p_1}^2 + {p_2}^2 = a^2$.

108. (a) : $2x + y = 2$ ...(i), $x + 2y = 2$ ...(ii)

Solving (i) and (ii), we get

$$x = \frac{2}{3}, y = \frac{2}{3}$$

$$d = \sqrt{\left(1 - \frac{2}{3}\right)^2 + \left(2 - \frac{2}{3}\right)^2} = \frac{\sqrt{17}}{3}$$

109. (e) : The required line is $4x - y = k$ which passes through $(5, 0)$.

$\Rightarrow$ $20 - 0 = k \Rightarrow k = 20$

$\therefore$ Required equation is $4x - y = 20$.

or, $\frac{x}{5} - \frac{y}{20} = 1$ or, $\frac{x}{5} + \frac{y}{-20} = 1$

$\therefore$ It meets the y-axis at $(0, -20)$.

110. (b) : We have, x intercept $= \frac{k}{5}$ and y intercept $= \frac{k}{1}$

$$\text{Area} = \left|\frac{1}{2} \times \frac{k}{5} \times \frac{k}{1}\right| = \left|\frac{k^2}{10}\right| = 10$$

$\Rightarrow$ $k = \pm 10$

111. (d) : We have $x + y = 6$ ...(i), $x + 2y = 4$ ...(ii)

Solving (i) and (ii), we get

$y = -2, x = 8 \Rightarrow (8, -2)$

$\therefore$ centre : $(8, -2)$

$$r = \sqrt{(8-6)^2 + (-2-2)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

112. (d) : We have, $2g = -6 \Rightarrow g = -3$,

$2f = 2 \Rightarrow f = 1$, $c = -28$

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{9 + 1 + 28} = \sqrt{38}$$

centre : $(h, k) = (-g, -f) = (3, -1)$

$\therefore$ $x = h + r\cos\theta$, $y = k + r\sin\theta$ are parametric form.

$\Rightarrow$ $x = 3 + \sqrt{38}\cos\theta$, $y = -1 + \sqrt{38}\sin\theta$

113. (c)

114. (b) : $(5, 0)$ —— $P(x, y)$ —— $(10\cos\theta, 10\sin\theta)$ (2 : 3)

$$x = \frac{20\cos\theta + 15}{5}, \quad y = \frac{20\sin\theta + 0}{5}$$

$\Rightarrow$ $5x - 15 = 20\cos\theta$, $5y = 20\sin\theta$

$$\Rightarrow \quad \left[\frac{5(x-3)}{20}\right]^2 + \left(\frac{y}{4}\right)^2 = 1$$

$\Rightarrow$ $(x - 3)^2 + y^2 = 16$ is the required equation of circle.

115. (c) : General equation of the circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

It passes through $(0, 0) \Rightarrow c = 0$

it passes through $(a, 0) \Rightarrow 2g = -a$

it passes through $(0, a) \Rightarrow 2f = -a$

$\therefore$ Required equation of the circle is

$x^2 + y^2 - ax - ay = 0$

$\Rightarrow$ $x^2 + y^2 = a(x + y)$

116. (d) : We have, $a = 3$, $b = 3$

so, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1+1} = \sqrt{2}$

Distance between the directrices $= \frac{2a}{e}$

$$= \frac{2 \cdot 3}{\sqrt{2}} = 3\sqrt{2}$$

117. (e)

118. (c) : We have, $a = 3, \frac{2b^2}{a} = \frac{16}{9} \quad \Rightarrow \quad b^2 = \frac{8}{3}$

$\Rightarrow$ Required equation of ellipse is

$$\frac{x^2}{9} + \frac{y^2}{\frac{8}{3}} = 1 \Rightarrow \frac{x^2}{9} + \frac{3y^2}{8} = 1$$

119. (b) : $a = 5$, $b = 4$

Now, 4 max $\{CP\}$ + 5min $\{CP\}$

$= (4 \times 5) + (5 \times 4) = 40$

120. (a)

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

1. If z_1, z_2, z_3 are complex numbers such that

$|z_1| = |z_2| = |z_3| = 1$ and $\frac{z_1^2}{z_2 z_3} + \frac{z_2^2}{z_3 z_1} + \frac{z_3^2}{z_1 z_2} = -1$,

then find the value of $|z_1 + z_2 + z_3|$

–Abhijit Kr. (Bhubaneshwar)

Ans. Let $z = z_1 + z_2 + z_3$. Then

$$\bar{z} = \bar{z}_1 + \bar{z}_2 + \bar{z}_3 = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}$$

$$= \frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 z_2 z_3}$$

$\Rightarrow \bar{z}\, b = z_1 z_2 + z_2 z_3 + z_3 z_1$, where $b = z_1 z_2 z_3$.

Now,

$$\frac{z_1^2}{z_2 z_3} + \frac{z_2^2}{z_3 z_1} + \frac{z_3^2}{z_1 z_2} = -1 \Rightarrow z_1^3 + z_2^3 + z_3^3 = -z_1 z_2 z_3$$

$\Rightarrow z_1^3 + z_2^3 + z_3^3 - 3z_1 z_2 z_3 = -4b$

$\Rightarrow (z_1 + z_2 + z_3)[(z_1 + z_2 + z_3)^2 - 3(z_1 z_2 + z_2 z_3 + z_3 z_1)] = -4b$

$\Rightarrow z(z^2 - 3\bar{z}\, b) = -4b$

$\Rightarrow z^3 - 3|z|^2 b + 4b = 0$

$\Rightarrow z^3 = (3|z|^2 - 4)b$

$\Rightarrow |z|^3 = |3|z|^2 - 4|$ (since $|b| = |z_1 z_2 z_3| = 1$)

Case 1 : Suppose that $3|z|^2 \geq 4$. Then

$|z|^3 = 3|z|^2 - 4$

$\Rightarrow |z|^3 - 3|z|^2 + 4 = 0$

$\Rightarrow (|z| - 2)(|z|^2 - |z| - 2) = 0$

$\Rightarrow (|z| - 2)(|z| - 2)(|z| + 1) = 0$

$\Rightarrow |z| = 2$

Case 2 : Suppose that $3|z|^2 < 4$. Then

$|z|^3 = |3|z|^2 - 4| = 4 - 3|z|^2$

$\Rightarrow |z|^3 + 3|z|^2 - 4 = 0$

$\Rightarrow (|z| - 1)(|z|^2 + 4|z| + 4) = 0$

$\Rightarrow (|z| - 1)(|z| + 2)^2 = 0 \Rightarrow |z| = 1$.

2. Find the general solution of the equation $\sin^4 2x + \cos^4 2x = \sin 2x \cos 2x$.

–Anand (W.B.)

Ans. We have,

$(\sin^2 2x + \cos^2 2x)^2 - 2\sin^2 2x \cos^2 2x = \sin 2x \cos 2x$

$\Rightarrow 2\sin^2 2x \cos^2 2x + \sin 2x \cos 2x - 1 = 0$

Put $\sin 2x \cos 2x = t$. Therefore, we have

$2t^2 + t - 1 = 0$

$\Rightarrow (2t - 1)(t + 1) = 0 \Rightarrow t = \frac{1}{2}, -1$

Case 1 : When $t = -1$, we have

$\sin 2x \cos 2x = -1$

$\Rightarrow \sin 4x = -2$, which is impossible.

Therefore $t = -1$ is to be rejected.

Case 2 : When $t = 1/2$, we have

$\sin 4x = 1$

$\Rightarrow 4x = 2n\pi + (\pi/2)$

$\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8} = (4n + 1)\frac{\pi}{8}$

3. A person has to go through three successive tests. The probability of his passing first test is p. If he fails in one of the tests, then the probability of his passing next test is $p/2$, otherwise it remains the same. For selection, the person must pass atleast two tests. Find the probability that the person has to be selected.

–Devika (Kochi)

Ans. Let E_i $(i = 1, 2, 3)$ be the event of the person passing the i^{th} test and E is the event that he is selected.

Then

$E = (E_1 \cap E_2) \cup (E_1 \cap \bar{E}_2 \cap E_3) \cup (\bar{E}_1 \cap E_2 \cap E_3)$

Therefore

$$P(E) = P(E_1)P(E_2 \mid E_1) + P(E_1)P(\bar{E}_2 \mid E_1)P(E_3 \mid \bar{E}_2) + P(\bar{E}_1)P(E_2 \mid \bar{E}_1)P(E_3 \mid E_2)$$

$$= p \cdot p + p(1 - p)\frac{p}{2} + (1 - p)\frac{p}{2} \cdot p$$

$= 2p^2 - p^3$.

4. If $\vec{x} \times \vec{a} + (\vec{x} \cdot \vec{b})\vec{c} = \vec{d}$, then prove that

$\vec{x} = \lambda\vec{a} + \vec{a} \times \frac{\vec{a} \times (\vec{d} \times \vec{c})}{(\vec{a} \cdot \vec{c})\vec{a}^2}$, where λ is a scalar.

–Rajkumar (Patna)

Ans. Here $\vec{x} \times \vec{a} + (\vec{x} \cdot \vec{b})\vec{c} = \vec{d}$

$\therefore \{\vec{x} \times \vec{a} + (\vec{x} \cdot \vec{b})\vec{c}\} \times \vec{c} = \vec{d} \times \vec{c}$

$\Rightarrow (\vec{x} \times \vec{a}) \times \vec{c} = \vec{d} \times \vec{c}$ $[\because \vec{c} \times \vec{c} = \vec{0}]$

$\Rightarrow \quad (\vec{x}\cdot\vec{c})\vec{a}-(\vec{a}\cdot\vec{c})\vec{x}=\vec{d}\times\vec{c}$

$\therefore \quad \vec{a}\times\{(\vec{x}\cdot\vec{c})\vec{a}-(\vec{a}\cdot\vec{c})\vec{x}\}=\vec{a}\times(\vec{d}\times\vec{c})$

$\Rightarrow \quad -(\vec{a}\cdot\vec{c})(\vec{a}\times\vec{x})=\vec{a}\times(\vec{d}\times\vec{c}) \quad \{\because \quad \vec{a}\times\vec{a}=\vec{0}\}$

$\Rightarrow \quad \vec{x}\times\vec{a}=\dfrac{\vec{a}\times(\vec{d}\times\vec{c})}{\vec{a}\cdot\vec{c}}$

$\Rightarrow \quad \vec{a}\times(\vec{x}\times\vec{a})=\vec{a}\times\dfrac{\vec{a}\times(\vec{d}\times\vec{c})}{\vec{a}\cdot\vec{c}}$

$\Rightarrow \quad (\vec{a}\cdot\vec{a})\vec{x}-(\vec{a}\cdot\vec{x})\vec{a}=\vec{a}\times\dfrac{\vec{a}\times(\vec{d}\times\vec{c})}{\vec{a}\cdot\vec{c}}$

$\Rightarrow \quad \vec{a}^2\vec{x}=(\vec{a}\cdot\vec{x})\vec{a}+\vec{a}\times\dfrac{\vec{a}\times(\vec{d}\times\vec{c})}{\vec{a}\cdot\vec{c}}$

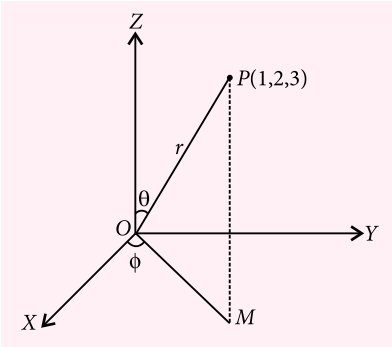
$\therefore \quad \vec{x}=\dfrac{(\vec{a}\cdot\vec{x})}{\vec{a}^2}\vec{a}+\vec{a}\times\dfrac{\vec{a}\times(\vec{d}\times\vec{c})}{(\vec{a}\cdot\vec{c})\vec{a}^2}$

$= \lambda\vec{a}+\vec{a}\times\dfrac{\vec{a}\times(\vec{d}\times\vec{c})}{(\vec{a}\cdot\vec{c})\vec{a}^2}$,

where $\lambda = \dfrac{\vec{a}\cdot\vec{x}}{\vec{a}^2}$ = scalar.

5. Let PM be the perpendicular from the point $P(1, 2, 3)$ to the xy plane. If OP makes an angle θ with the positive direction of the z-axis and OM makes an angle ϕ with the positive direction of the x-axis, where O is the origin, then find θ and ϕ.

–Rahil (U.P.)



Ans. We know that if $P = (x, y, z)$ then

$x = r\sin\theta\cdot\cos\phi,\ y = r\sin\theta\cdot\sin\phi,\ z = r\cos\theta$

$\therefore \quad 1 = r\sin\theta\cdot\cos\phi,\ 2 = r\sin\theta\cdot\sin\phi,\ 3 = r\cos\theta$

$\Rightarrow \quad 1^2 + 2^2 = r^2\sin^2\theta\cdot\cos^2\phi + r^2\sin^2\theta\cdot\sin^2\phi$

$= r^2\sin^2\theta(\cos^2\phi + \sin^2\phi) = r^2\sin^2\theta$

$\Rightarrow \quad 5 = r^2\sin^2\theta \quad \therefore \quad r\sin\theta = \sqrt{5}$

(clearly, θ and ϕ are acute)

$\therefore \quad \dfrac{r\sin\theta}{r\cos\theta}=\dfrac{\sqrt{5}}{3}$ or $\tan\theta = \dfrac{\sqrt{5}}{3}$

$\therefore \quad \theta = \tan^{-1}\left(\dfrac{\sqrt{5}}{3}\right)$

Also, $\dfrac{1}{2}=\dfrac{r\sin\theta\cdot\cos\phi}{r\sin\theta\cdot\sin\phi} \Rightarrow \cot\phi = \dfrac{1}{2} \Rightarrow \tan\phi = 2.$

$\therefore \quad \phi = \tan^{-1}2.$

6. Find the value of $\displaystyle\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin 2x}dx$ ***–Vikas (M.P.)***

Ans. Let $I = \displaystyle\int_0^{\pi/4}\frac{d(\sin x-\cos x)}{9-16(1-\sin 2x)+16}$

$= \displaystyle\int_0^{\pi/4}\frac{d(\sin x-\cos x)}{25-16(\sin x-\cos x)^2}$

$= \dfrac{1}{16}\displaystyle\int_0^{\pi/4}\frac{d(\sin x-\cos x)}{\left(\frac{5}{4}\right)^2-(\sin x-\cos x)^2}$

Put $\sin x - \cos x = z$; then

$x = 0 \Rightarrow z = -1;\ x = \dfrac{\pi}{4} \Rightarrow z = 0$

$\therefore \quad I = \dfrac{1}{16}\displaystyle\int_{-1}^{0}\frac{dz}{\left(\frac{5}{4}\right)^2-z^2} = \frac{1}{16}\cdot\left[\frac{1}{2\cdot\frac{5}{4}}\log\frac{\frac{5}{4}+z}{\frac{5}{4}-z}\right]_{-1}^{0}$

$= \dfrac{1}{40}\left[\log\dfrac{5+4z}{5-4z}\right]_{-1}^{0} = \dfrac{1}{40}\left\{\log 1-\log\dfrac{1}{9}\right\}$

$= \dfrac{1}{40}\log 9 = \dfrac{1}{20}\log 3.$

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Math Archives 10 Best Problems

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE (Main & Advanced) Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE (Main & Advanced). In every issue of MT, challenging problems are offered with detailed solution. The readers' comments and suggestions regarding the problems and solutions offered are always welcome.

1. If a, b, c, d are positive, then $\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx}=$

(a) $e^{d/b}$ (b) $e^{c/a}$

(c) $e^{\frac{(c+d)}{(a+b)}}$ (d) e

2. $\lim_{x\to 1}\frac{\sqrt{1+\cos 2(x-1)}}{x-1}$

(a) exists and it equals $\sqrt{2}$

(b) exists and it equals $-\sqrt{2}$

(c) does not exist because $x - 1 \to 0$

(d) does not exist because L.H.L is not equal to R.H.L

3. The function $\frac{\log(1+ax)-\log(1-bx)}{x}$ is not defined at $x = 0$. The value which should be assigned to f at $x = 0$ so that it is continuous at $x = 0$ is

(a) $a - b$ (b) $1 + b$

(c) $\log a + \log b$ (d) none of these

4. If a line OP through the origin O makes angle α, 45° and 60° with x, y and z axes respectively, then the direction cosines of OP are

(a) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$ (b) $\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$

(c) $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$ (d) none of these

5. If O is the origin and the line OP of length r makes an angle α with x-axis and lies in the xz plane, then the coordinates of P are

(a) $(r\cos\alpha, 0, r\sin\alpha)$ (b) $(0, 0, r\sin\alpha)$

(c) $(0, 0, r\cos\alpha)$ (d) $(r\cos\alpha, 0, 0)$

6. Evaluate : $\lim_{x\to\pi/2}\frac{\cos x}{(1-\sin x)^{2/3}}$.

7. Evaluate : $\lim_{x\to\infty}\left(\sin\frac{1}{x}+\cos\frac{1}{x}\right)^{x}$

8. If f and g are differentiable at $a \in R$ such that $f(a) = g(a) = 0$ and $g'(a) \neq 0$, then show that $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{f'(a)}{g'(a)}$.

9. Show that the function g, defined by $g(x) = \sin\alpha + \cos\alpha - 1$, $\alpha=\sin^{-1}\sqrt{\{x\}}$, $\{\ .\ \}$ denotes fractional part function, is an even function.

10. Evaluate : $\lim_{x\to-\infty}\frac{\sqrt{2x^2+1}}{4x+1}$

SOLUTIONS

1. (a) : $\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx}$

$= \lim_{x\to\infty}\left[\left(1+\frac{1}{a+bx}\right)^{a+bx}\right]^{\frac{c+dx}{a+bx}} = e^{\lim_{x\to\infty}\frac{\frac{c}{x}+d}{\frac{a}{x}+b}}$

Solution Sender of Maths Musing

SET-149

1. Khokon Kumar Nandi - W.B.
2. Gajula Ravinder - Karimnagar (T.S.)
3. Gouri Sankar Adhikari - W.B.

SET-148

1. Raghav sen, Snighda, N. Jayanthi - Hyderabad
2. Gouri Sankar Adhikari - W.B.
3. Gajula Ravinder - Karimnagar (T.S.)
4. Anubhav Banerjee - Kolkata

By : **Prof. Shyam Bhushan,** Director, Narayana IIT Academy, Jamshedpur. Mob. : 09334870021

2. **(c) :** Put $x - 1 = t$

$$\lim_{x\to 1}\frac{\sqrt{1+\cos 2(x-1)}}{x-1} = \lim_{t\to 0}\frac{\sqrt{2}|\cos t|}{t}$$

$$= \lim_{t\to 0}\frac{\sqrt{2}\cos t}{t}$$ does not exist because

$t \to 0 \Rightarrow (x - 1) \to 0$

3. **(d) :** For continuity at $x = 0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x\to 0}\frac{a\cdot\log(1+ax)}{ax} + \frac{b\cdot\log(1-bx)}{-bx} = f(0)$$

$$\Rightarrow a + b = f(0)$$

4. **(c) :** Direction cosines of OP are $\cos\alpha$, $\cos 45°$, $\cos 60°$

Now $\cos^2\alpha + \cos^2 45° + \cos^2 60° = 1$

$$\Rightarrow \cos^2\alpha + \left(\frac{1}{2}\right) + \left(\frac{1}{4}\right) = 1 \Rightarrow \cos^2\alpha = \frac{1}{4}$$

$$\Rightarrow \cos\alpha = \frac{1}{2}$$

5. **(a) :** Let the coordinates of P be (x, y, z)

Since OP lies in xz plane and makes an angle α with the x–axis, it makes angle $\left(\frac{\pi}{2} - \alpha\right)$ with z–axis and $\frac{\pi}{2}$ with y–axis. So,

$x = r\cos\alpha, y = r\cos\frac{\pi}{2}, z = r\cos\left(\frac{\pi}{2} - \alpha\right)$ are the required coordinates and therefore are $(r\cos\alpha, 0, r\sin\alpha)$

6. $$\lim_{x\to\pi/2}\frac{\cos x}{(1-\sin x)^{2/3}} = \lim_{t\to 0}\frac{\sin t}{(1-\cos t)^{2/3}}$$

$$= \lim_{t\to 0}\frac{2\sin(t/2)\cos(t/2)}{(2\sin^2(t/2))^{2/3}}$$

$$= \lim_{t\to 0} 2^{1/3}\frac{\cos(t/2)}{(\sin(t/2))^{1/3}}.$$

Limit value is ∞ as $t \to 0^+$ and limit value is $-\infty$ as $t \to 0^-$. Thus, limit does not exist.

7. $$\lim_{x\to\infty}\left(\sin\frac{1}{x} + \cos\frac{1}{x}\right)^x = e^{\lim\limits_{x\to\infty} x\left(\sin\frac{1}{x} + \cos\frac{1}{x} - 1\right)}$$

$$= e^{\lim\limits_{x\to\infty}\frac{\sin(1/2x)}{1/2x}\left(\cos\frac{1}{2x} - \sin\frac{1}{2x}\right)} = e$$

8. $$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}} = \frac{f'(a)}{g'(a)},$$

as $f(a) = g(a) = 0$ and $g'(a) \neq 0$.

9. $$g(x) = \sin(\sin^{-1}\sqrt{\{x\}}) + \cos(\sin^{-1}\sqrt{\{x\}}) - 1$$

$$= \sqrt{\{x\}} + \cos(\cos^{-1}\sqrt{1-\{x\}}) - 1$$

$$= \sqrt{\{x\}} + \sqrt{1-\{x\}} - 1.$$

If $x \in I$, then $\{x\} = 0 \Rightarrow g(x) = 0$

$\Rightarrow g(x) = g(-x)$

If $x \notin I$, then $\{-x\} = 1 - \{x\}$

$\Rightarrow g(-x) = \sqrt{1-\{x\}} + \sqrt{\{x\}} - 1 = g(x)$

Hence g is an even function.

10. $$\lim_{x\to-\infty}\frac{\sqrt{2x^2+1}}{4x+1} = \lim_{x\to-\infty} -\frac{\sqrt{2+\frac{1}{x^2}}}{4+\frac{1}{x}}, \text{ as } x<0$$

$$= -\frac{1}{2\sqrt{2}}.$$

■■

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QUANTITATIVE APTITUDE

PRACTICE PAPER

1. What least value must be given to * so that the number 97215 * 6 is divisible by 11 ?

(a) 1 (b) 2 (c) 3 (d) 5

2. The traffic lights at three different road crossings change after every 48 secs, 72 secs and 108 secs respectively. If they all change simultaneously at 8 : 20 : 00 hrs; then they will again change simultaneously at

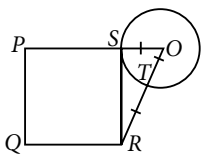
(a) 8 : 27 : 12 hrs (b) 8 : 27 : 24 hrs

(c) 8 : 27 : 36 hrs (d) 8 : 27 : 48 hrs

3. The difference between a discount of 40% on ₹ 500 and two successive discounts of 36% and 4% on the same amount is

(a) ₹0 (b) ₹2 (c) ₹1.93 (d) ₹7.20

4. *PQRS* is a square. *SR* is a tangent (at point *S*) to the circle with centre *O* and *TR* = *OS*. Then, the ratio of area of the circle to the area of the square is



(a) π : 3 (b) 11 : 7 (c) 3 : π (d) 7 : 11

5. *A* and *B* walk from *X* to *Y*, a distance of 27 km at 5 km/hr and 7 km/hr respectively. *B* reaches *Y* and immediately turns back meeting *A* at *Z*. What is the distance from *X* to *Z* ?

(a) 25 km (b) 22.5 km (c) 24 km (d) 20 km

6. The ratio of the number of boys and girls in a school is 3 : 2. If 20% of the boys and 25% of the girls are scholarship holders, what percentage of the students does not get the scholarship ?

(a) 56% (b) 70% (c) 78% (d) 80%

7. *A* can lay railway track between two given stations in 16 days and *B* can do the same job in 12 days. With the help of *C*, they did the job in 4 days only. Then, *C* alone can do the job in

(a) $9\frac{1}{5}$ days (b) $9\frac{2}{5}$ days

(c) $9\frac{3}{5}$ days (d) 10 days

8. Mr. Thomas invested an amount of ₹ 13,900 divided in two different schemes A and B at the simple interest of rate 14% p.a. and 11% p.a. respectively. If the total amount of simple interest earned in 2 years be ₹3508, what was the amount invested in Scheme B ?

(a) ₹6400 (b) ₹6500

(c) ₹7500 (d) None of these

9. If the radius of circle is diminished by 10%, then its area is diminished by

(a) 10% (b) 19% (c) 20% (d) 36%

10. Peter bought an item at 20% discount on its original price. He sold it with 40% increase on the price he bought it. The new sale price is by what percent more than the original price ?

(a) 7.5% (b) 8% (c) 10% (d) 12%

11. In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither blue nor green ?

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{7}{19}$ (d) $\frac{8}{21}$

12. $\left[\frac{1}{(\log_a bc)+1}+\frac{1}{(\log_b ca)+1}+\frac{1}{(\log_c ab)+1}\right]$ is equal to

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) 3

13. The income of a broker remains unchanged though the rate of commission is increased from 4% to 5%. The percentage of slump in business is

(a) 1% (b) 8% (c) 20% (d) 80%

14. The value of the expression 1014×986 is

(a) 998924 (b) 999864

(c) 999804 (d) 996724

15. The dimensions of an open box are 52 cms, 40 cms, and 26 cms. Its thickness is 1 cm. If 1 cubic cm. of the metal used in the box weighs 1 gm, then the weight of the box is

(a) 8.48 kg (b) 6.58 kg

(c) 7.28 kg (d) 658 gm

16. The unit digit in the sum $(264)^{102} + (264)^{103}$ is

(a) 0 (b) 4 (c) 6 (d) 8

17. The least number, which when increased by 1 is exactly divisible by 12, 18, 24, 32, 40 is

(a) 1439 (b) 1440 (c) 1449 (d) 1459

18. One litre of water weighs 1 kg. How many cubic millimetres of water will weigh 0.1 gram ?

(a) 10 (b) 100 (c) 0.1 (d) 1

19. A man has ₹480 in the denominations of one-rupee notes, five-rupee notes and ten-rupee notes. The number of notes are equal. What is the total number of notes he has ?

(a) 45 (b) 60 (c) 75 (d) 90

20. 5 mangoes and 4 oranges cost as much as 3 mangoes and 7 oranges. What is the ratio of the cost of one mango to that of one orange?

(a) 4 : 3 (b) 1 : 3 (c) 3 : 2 (d) 5 : 2

SOLUTIONS

1. (c) : For the number to be divisible by 11, the difference between the sum of digits at odd places and that of at even places should be either 0 or divisible by 11.

$\therefore$ (9 + 2 + 5 + 6) – (7 + 1 + *) = 14 – *

So, * = 3

2. (a) : Interval of change

= (L.C.M. of 48, 72, 108) = 432secs

So, the lights will change after every 432 seconds *i.e.* 7 mins 12 secs.

So, the next simultaneous change will take place at 8 : 27 : 12 hrs.

3. (d) : Price after 40% discount = 60% of ₹500 = ₹300.

Price after 36% discount = 64% of ₹500 = ₹320.

Price after next 4% discount = 96% of ₹320 = ₹307.20.

$\therefore$ Difference in two prices = ₹7.20.

4. (a) : Let the radius of the circle be r

Then area of the circle is πr^2.

Now $OR = OT + TR = r + r = 2r$

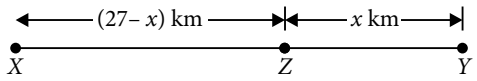
$[\because TR = OS = r$ (given)]

Now, In ΔOSR by pythagoras theorem, $SR = \sqrt{3}r$

So, area of square $PQRS = 3r^2$

$\therefore$ Required ratio $= \frac{\pi r^2}{3r^2} = \frac{\pi}{3}$ or $\pi : 3$

5. (b) : Time taken by A in covering $(27 - x)$ km is same as time taken by B in covering $(27 + x)$ km.



$\Rightarrow \frac{(27-x)}{5} = \frac{(27+x)}{7}$

$\Rightarrow x = \frac{27}{6}$

$\therefore XZ = 27 - \frac{27}{6} = 27 \times \frac{5}{6} = 22.5$ km.

6. (c) : Let boys = $3x$ and girls = $2x$.

Number of those who do not get scholarship

= (80% of $3x$) + (75% of $2x$)

$= \left(\frac{80}{100} \times 3x\right) + \left(\frac{75}{100} \times 2x\right) = \frac{39x}{10}$.

$\therefore$ Required percentage $= \left(\frac{39x}{10} \times \frac{1}{5x} \times 100\right)\% = 78\%$.

7. (c) : (A + B + C)'s 1 day work $= \frac{1}{4}$,

A's 1 day work $= \frac{1}{16}$, B's 1 day work $= \frac{1}{12}$.

$\therefore$ C's 1 day work $= \frac{1}{4} - \left(\frac{1}{16} + \frac{1}{12}\right) = \left(\frac{1}{4} - \frac{7}{48}\right) = \frac{5}{48}$.

So, C alone can do the work in $\frac{48}{5} = 9\frac{3}{5}$ days.

8. (a) : Let the sum invested in Scheme A be ₹x and that in Scheme B be ₹$(13900 - x)$.

Then, $\left(\frac{x \times 14 \times 2}{100}\right) + \left[\frac{(13900 - x) \times 11 \times 2}{100}\right] = 3508$

$\Rightarrow$ $28x - 22x = 350800 - (13900 \times 22)$

$\Rightarrow$ $6x = 45000 \Rightarrow x = 7500$.

So, sum invested in Scheme B = ₹$(13900 - 7500)$

= ₹6400.

9. (b) : Let the original radius be R units

New radius = (90% of R) $= \left(\frac{90}{100} \times R\right) = \frac{9R}{10}$ units.

Original area = πR^2.

$\therefore$ Diminished area

$= \left[\pi R^2 - \pi\left(\frac{9R}{10}\right)^2\right]$ sq. units $= \left[\left(1 - \frac{81}{100}\right)\pi R^2\right]$ sq. units

$= \left(\frac{19}{100}\pi R^2\right)$ sq. units.

Decrease % $= \left(\frac{19\pi R^2}{100} \times \frac{1}{\pi R^2} \times 100\right)\% = 19\%$.

10. (d) : Let the original price be ₹100

Then, C.P = ₹80.

S.P. = 140% of ₹80 = ₹$\left(\frac{140}{100} \times 80\right) =$ ₹112.

$\therefore$ Required percentage = $(112 - 100)\% = 12\%$.

11. (d) : Total number of balls = $(8 + 7 + 6) = 21$.

Let E be the event that the ball drawn is neither blue nor green

i.e; E be the event that the ball drawn is red.

$\therefore$ $n(E) = 8$

$\therefore$ $P(E) = \frac{8}{21}$.

12. (a) : Given expression can be written as;

$$\frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$$

$$= \frac{1}{\log_a (abc)} + \frac{1}{\log_b (abc)} + \frac{1}{\log_c (abc)}$$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c = \log_{abc} (abc) = 1$.

13. (c) : Suppose the business value changes from x to y.

4% of x = 5% of $y \Rightarrow \frac{4}{100}x = \frac{5}{100}y \Rightarrow y = \frac{4}{5}x$.

$\therefore$ Change in business $= \left(x - \frac{4}{5}x\right) = \frac{x}{5}$.

Percentage of slump $= \left(\frac{x}{5} \times \frac{1}{x} \times 100\right)\% = 20\%$.

14. (c) : $1014 \times 986 = (1000 + 14)(1000 - 14)$

$= (1000)^2 - (14)^2 = (1000000 - 196) = 999804$.

15. (b) : Volume of metal

$= (52 \times 40 \times 26 - 50 \times 38 \times 25)$ cu. cm.

$= (54080 - 47500)$ cu. cm. $= 6580$ cu. cm.

$\therefore$ Weight of metal = 6580 gm = 6.58 kg.

16. (a) : $(264)^{102} + (264)^{103} = (264)^{102} \times [1 + 264]$

$= (264)^{102} \times 265$

Unit digit in $(264)^4$ is 6

Unit digit in $[(264)^4]^{25}$ = Unit digit in $6^{25} = 6$

Unit digit in $(264)^{100} \times (264)^2$ is 6

Unit digit in $(264)^{102} \times 265$ = unit digit in $6 \times 5 = 0$

17. (a) :

2	12, 18, 24, 32, 40
2	6, 9, 12, 16, 20
3	3, 9, 6, 8, 10
2	1, 3, 2, 8, 10
	1, 3, 1, 4, 5

$\therefore$ L.C.M. of 12, 18, 24, 32, 40 = 1440

$\therefore$ Required number = $(1440 - 1) = 1439$.

18. (b) : 1000 cu. cm weighs 1000 gms.

i.e. 1000 gms is the weight of (1000×1000) cu. mm

0.1 gm is the weight of $\left(\frac{1000 \times 1000}{1000} \times 0.1\right)$ cu. mm

= 100 cu. mm.

19. (d) : Let the number of each type of notes be x.

Then, $x + 5x + 10x = 480 \Rightarrow x = 30$.

$\therefore$ Total number of notes = 30 + 30 + 30 = 90.

20 (c) : Let cost of each mango be x paise and that of each orange be y paise, then $5x + 4y = 3x + 7y$

$\Rightarrow 2x = 3y \Rightarrow x : y = 3 : 2$.

MATHS MUSING

SOLUTION SET-149

1. (c) : Consider the blocks, 1; 2, 1; 2, 2, 1; 2, 2, 2, 1;..... while the n^{th} block consists of $(n-1)2$'s and one 1.
The sum of terms in n blocks is $1 + 3 + 5 + ... + (2n - 1) = n^2$
The number of terms in n blocks is $1 + 2 + 3 + ...+ n = \frac{n(n+1)}{2}$
Taking $n = 63$, sum of terms $= 63^2 = 3969$ and number of terms $= 2016$. $\therefore$ The sum of 2015 terms is 3968

2. (d) : $f(a - x) = f(a + x)$ ...(i)
$f(b - x) = f(b + x)$ ...(ii)
Let $x = a$ in (i) and $x = b$ in (ii)
$\Rightarrow f(0) = f(2a) = f(2b) \Rightarrow f\{2b + 2(a - b)\} = f(2a)$.
$\therefore$ $f(x)$ is periodic with period $2(a-b)$

3. (b) : $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$
The centroid $G = \left(\frac{\Sigma x_i}{3}, \frac{\Sigma y_i}{3}\right)$.

$$PG^2 = \left(\frac{x_1 + x_2 + x_3}{3} - x\right)^2 + \left(\frac{y_1 + y_2 + y_3}{3} - y\right)^2$$

$\Rightarrow 9\,PG^2 = [3(PA^2 + PB^2 + PC^2) - (a^2 + b^2 + c^2)]$
$= 210 - 110$
$\therefore PG = \left(\frac{10}{3}\right)$. P describes a circle of radius $\left(\frac{10}{3}\right)$

4. (a) : $(1+x)^{1/x} = e^{1/x\, ln(1+x)}$

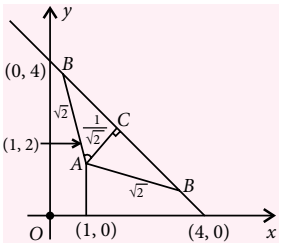
$$(1+x)^{1/x} - e + \frac{ex}{2} = e^{\left(1 - \frac{x}{2} + \frac{x^2}{3} - .....\right)} - e + \frac{ex}{2}$$

$$= e \cdot e^{\left(-\frac{x}{2} + \frac{x^2}{3} - .....\right)} - e + \frac{ex}{2}$$

$$= 1 + \left(-\frac{x}{2} + \frac{x^2}{3}\right) + \frac{\left(-\frac{x}{2} + \frac{x^2}{3}\right)^2}{2} = ex^2\left(\frac{1}{3} + \frac{1}{8}\right)$$

$\therefore$ Limit $= \frac{1}{3} + \frac{1}{8} = \frac{11}{24}$

5. (c), (d) : The distance of A from the line $x + y = 4$ is $\frac{1}{\sqrt{2}} = AC$
$\Rightarrow \angle BAC = 60°$
Slope of angle is $\tan(45° \pm 60°)$
$= \tan 105°$, $\tan(-15)°$
$= -2 + \sqrt{3}$, $-2 - \sqrt{3}$

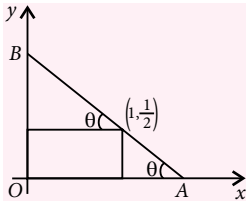


6. (a), (b) : Tangent $y = mx + \frac{1}{m}$
$x = \frac{t}{\sqrt{2}}$, $y = \frac{t^2}{2\sqrt{2}}$ lies on $x^2 = \sqrt{2}y$.
Normal is $y = -\frac{x}{t} + \frac{1}{\sqrt{2}} + \frac{t^2}{2\sqrt{2}}$ $\therefore$ $t = -\frac{1}{m}$,
$t^2 + 2\sqrt{2}\,t + 2 = 0 \Rightarrow t = -\sqrt{2}$
$m = \frac{1}{\sqrt{2}}$, L is $x - \sqrt{2}\,y + 2 = 0$
Distance of (0, 0) is $\frac{2}{\sqrt{3}}$.
L is also y-axis, giving distance 0.

7. (b) : $A = (at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$
$\Rightarrow t_1 t_2 = 2$ and $t_1 + t_2 + t = 0$
The orthocentre of ΔABC is given by $O(x, y)$
$x = a(t^2 - 6)$, $y = -at$.
Eliminating t, the locus is $y^2 = a(x + 6a)$.
L.R. $= a$

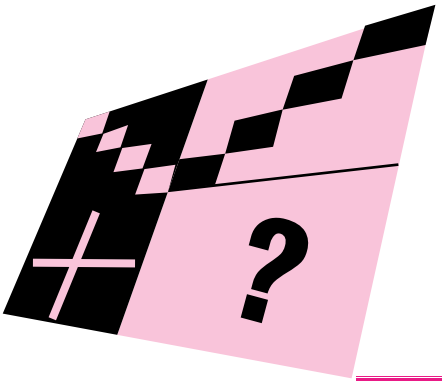
8. (a) : The centroid of ΔABC is $G\left(\frac{2a}{3}(t^2 - 2), 0\right)$
The circumcentre S divides OG in the ratio $3 : -1$.
$\therefore$ $S(x, y)$, $x = \frac{a}{2}(t^2 + 2)$, $y = \frac{at}{2}$.
Eliminating t, the locus is $y^2 = \frac{a}{2}(x - a)$, L.R. $= \frac{a}{2}$.

9. (5) : Perimeter $P = OA + OB + AB$
$OA = 1 + \frac{1}{2}\cot\theta$,
$OB = \frac{1}{2} + \tan\theta$
$AB = \frac{1}{2}\,\text{cosec}\theta + \sec\theta$



$\therefore$ $P = \frac{3}{2} + \frac{1}{2}(\cot\theta + \text{cosec}\theta) + \tan\theta + \sec\theta$
If $t = \tan\frac{\theta}{2}$, then $P = \frac{1}{2} + \frac{1}{2t} + \frac{2}{1-t} = f(t)$
$f'(t) = 0 \Rightarrow t = \frac{1}{3}$, $P_{min} = 5$

10. (b) : $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{21} + \frac{1}{24} + \frac{1}{28} = \frac{1}{8}$
$\therefore$ $r = 8$, $\Delta = \sqrt{r_1 r_2 r_3 r} = 336$
$s - a = \frac{\Delta}{r_1} = \frac{336}{21} = 16 \Rightarrow s - b = \frac{\Delta}{r_2} = \frac{336}{24} = 14$
$s - c = \frac{\Delta}{r_3} = \frac{336}{28} = 12$. Adding, $3s - (a + b + c) = s = 42$
$a = 26$, $b = 28$, $c = 30$.



Brain Teasers

DIRECTIONS (1 - 5) : Answer the questions on the basis of the information given below.

In one of my visits to a small but beautiful town, while I was out on a shopping spree at the supermarket, a little statue of Buddha took my fancy. When I opened my wallet to pay for it, I realized to my horror that I had run out of money. The woman at the cash counter told me where the nearest bank was but I reached there only to find that on that day of the week it was closed. Frustrated I drove back home and the first thing I asked my host was which place the supermarket, the department store, and the bank was open on which day of the week. She left me none the wiser when she started to tell me the curiosities of logic town (the town I was visiting).

In logic town the supermarket, the department store and the bank are open together on one day each week.

1. Each of the three places is open four days a week.
2. On Sunday all three places are closed.
3. None of the three places is open on three consecutive days.
4. On six consecutive days I observed that
   - the department store was closed on the first day
   - the supermarket was closed on the second day
   - the bank was closed on the third day
   - the supermarket was closed on the fourth day
   - the department store was closed on the fifth day
   - the bank was closed on the sixth day

1. Of the six consecutive days that my host spoke of, which day is first?

(a) Saturday (b) Friday
(c) Thursday (d) Tuesday

2. On which day of the week was I at the supermarket?

(a) Wednesday (b) Friday
(c) Monday (d) Saturday

3. Which of the statement(s) is/are correct?

I. Department store and bank are both open on Tuesday.
II. Bank and supermarket are both closed on Thursday.
III. Supermarket and department store are both open on Friday.
IV. Bank, department store and supermarket are all closed on Sunday.

(a) IV and I only (b) IV and II only
(c) IV and III only (d) All of these

4. If you arrived at logic town on Sunday then which place you could not be able to go to the next day?

(a) Bank
(b) Department store
(c) Supermarket
(d) Both bank and Supermarket.

5. On which day of the week are all three places in logic town open?

(a) Thursday (b) Friday
(c) Saturday (d) Tuesday

DIRECTIONS (6 & 7) : Answer the questions on the basis of the following information.

A student on holiday for d days observed that :

1. It rained seven times morning or afternoon.
2. When it rained in the afternoon, it was clear in the morning.
3. There were five clear afternoons.
4. There were six clear mornings.

6. How many days had clear mornings but rainy afternoons?

(a) 2 (b) 3 (c) 4 (d) 5

7. What is the value of d ?

(a) 10 (b) 9 (c) 8 (d) 7

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DIRECTIONS (8 – 12) : Answer the questions on the basis of the information given below.

The members of Gayan - a singing group - assembled on an estate one weekend to perform in two programmes.

A. Some of the members were to sing only in 'classical songs' programme and the rest of the members were to sing only in 'contemporary songs' programme.

B. A strange thing happened that weekend. Any statement made by a member who was to sing in 'classical songs' programme was true, and any statement made by a member who was to sing in 'contemporary songs' programme was false.

Before the programmes could start, a member of Gayan was murdered.

(1) Police then zeroed in on four suspects who made the following statements :

Moti : If the programme that the murderer was to sing in was, not the programme that I was to sing in, then the murderer's room is next to at least one room whose occupant was to sing in the programme that I was to sing in.

Neta : No two of us four who were to sing in the programme that I was to sing in, have adjacent rooms.

Onkar : The murderer was neither Neta nor Prem.

Prem : If the murderer was to sing in the programme that I was to sing in, then I am not the murderer.

(2) The four suspects had these connected rooms on the estate.

Moti	Neta	Onkar	Prem

8. With regard to the statements made by the suspects what can be said?

(a) Moti's hypothesis was false but conclusion was true.

(b) Moti's hypothesis was true but conclusion was false.

(c) Moti's hypothesis was false and conclusion was also false.

(d) Moti's hypothesis was true and conclusion was also true.

9. Who among the following lied?

(a) Moti (b) Onkar

(c) Prem (d) Both Onkar and Prem

10. Whom among the following told the truth?

(a) Onkar (b) Moti

(c) Prem (d) Both Moti and Prem

11. Which of the following combinations about Prem is true?

(a) Prem's hypothesis was false but conclusion was true.

(b) Prem's hypothesis was true but conclusion was false.

(c) Prem's hypothesis was true and conclusion was also true.

(d) Prem's hypothesis was false and conclusion was also false.

12. Who was the murderer?

(a) Onkar (b) Moti (c) Neta (d) Prem

SOLUTIONS

1. (a) 2. (c) 3. (d) 4. (a) 5. (b)

1-5 : If Sunday is the first of the six consecutive days mentioned, then from (1), (2) and (4), the supermarket is closed only on Sunday, Monday and Wednesday. This is impossible, from (3).

If Monday is the first of the six consecutive days mentioned, then from (2) and (4) at least one place is closed each day. This is impossible because all three places are open together on one day each week.

If Tuesday is the first of the six consecutive days mentioned, then from (1), (2) and (4) the department store is closed only on Tuesday, Saturday and Sunday. This is impossible from (3).

If Wednesday is the first of the six consecutive days mentioned, then from (1), (2) and (4) the bank is closed only on Sunday, Monday and Friday, and the supermarket is closed only on Sunday, Thursday and Saturday. This is impossible from (3).

If Thursday is the first of the six consecutive days mentioned then from (1), (2) and (4), the bank is closed only on Tuesday, Saturday and Sunday. This is impossible, from (3).

If Friday is the first of the six consecutive days mentioned, then from (1), (2) and (4) the supermarket is closed only on Monday, Saturday and Sunday. This is impossible, from (3).

So, Saturday is the first of the six consecutive days mentioned; then from (1), (2) and (4) (C stands for closed, O for open).

	Sun	Mon	Tue	Wed	Thurs	Fri	Sat
Bank	C	C	O	O	C	O	O
Department store	C	O	O	C	O	O	C
Supermarket	C	O	C	–	–	–	–

From the above table, Friday must be the day all the three places are open.

To complete the table : From (1) and (3), the supermarket cannot be closed on Wednesday or Saturday. So the supermarket must be closed on Thursday.

So the table looks like

	Sun	Mon	Tue	Wed	Thurs	Fri	Sat
Bank	C	C	O	O	C	O	O
Department store	C	O	O	C	O	O	C
Supermarket	C	O	C	–	C	O	–

Now as each of the three places is open four days a week, we have the complete table.

	Sun	Mon	Tue	Wed	Thurs	Fri	Sat
Bank	C	C	O	O	C	O	O
Department store	C	O	O	C	O	O	C
Supermarket	C	O	C	O	C	O	O

6. (c) **7. (b)**

6-7 : Make a tabular column to have a good idea.

	Rainy afternoon	**Clear afternoon**
Rainy morning	x	y
Clear morning	z	w

By the problem, $x + y + z + w = d$ ... (i)

As it rained seven times morning or afternoon

$\Rightarrow \quad x + y + z = 7$ ... (ii)

As there were five clear afternoons

$\Rightarrow \quad y + w = 5$ ... (iii)

As there were six clear mornings

$\Rightarrow \quad z + w = 6$ ... (iv)

When it rained in the afternoon, it was clear in the morning. $\Rightarrow x = 0$ ... (v)

With $x = 0$, equations (i) & (ii) reduces to

$y + z = 7$ ... (i)

$y + w = 5$ ... (iii)

$z + w = 6$ ... (iv)

Adding them $y + z + w = \frac{1}{2}(7 + 5 + 6) = 9$

$\therefore \quad y = (y + z + w) - (z + w) = 9 - 6 = 3$

Similarly, $z = 4$ and $w = 2$. $d = y + z + w = 9$.

8. (d) 9. (b) 10. (d) 11. (a) 12. (c)

8-12 : The following reasoning uses A, B and (1).

Suppose Moti lied : Then his hypothesis was true and his conclusion was false. Then the murderer told the truth and the murderer's room was not next to a liar's room. Then Moti was not the murderer, and from (2), Neta was not the murderer. Then either Onkar or Prem was the murderer.

(i) Suppose Prem was the murderer; then Prem told the truth and his conclusion was false; then his hypothesis was false; but, because he was the murderer, his hypothesis was true. This situation is impossible, so Onkar was the murderer.

N	O	P
F	T	T

Then Onkar told the truth, and Prem told the truth because his conclusion was true. Then, from (2), Neta didnot tell the truth, so Neta lied. Then from (2), because Onkar's room was next to a member who was to sing in either programme, Moti's conclusion was true. Then Moti told the truth. So if Moti lied, then he told the truth. This situation is impossible, so Moti told the truth.

M	N	O	P
T	L	T	

Then Neta did not tell the truth, from (2); so Neta lied (Recall that Neta's room is adjacent to Moti's). Then Onkar lied, from (2), because either Onkar and Neta lied, or Onkar and Prem both lied. Then either Neta or Prem was the murderer.

Suppose Prem was the murderer. Then Prem lied because his hypothesis was true and his conclusion was false. Then Moti's hypothesis was true and because it has already been established that Onkar lied, his conclusion was false. Then Moti lied. But it has already been established that Moti told the truth. This situation is impossible, so Prem was not the murderer. Then Neta was the murderer. Then Prem told the truth because his conclusion was true, Prem's hypothesis was false, while Moti's hypothesis and conclusion were both true.

We summarise,

(i) Moti told the truth, his hypothesis and conclusion were both true.

(ii) Neta was the murderer.

(iii) Prem told the truth, his hypothesis was false but conclusion true.

(iv) Onkar lied.

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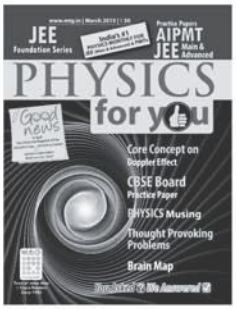

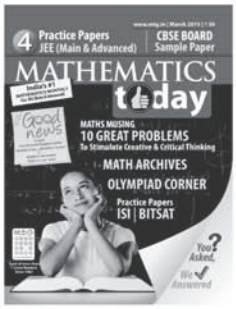

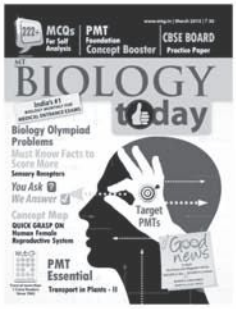

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